Majorana neutrino textures from numerical considerations.

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ZTPiCE

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Introduction.

- The presently available experimental results not only suggest a non-zero neutrino mass but also constrain the patterns of neutrino masses and mixing.
- These results make it now meaningful to confront various theoretical schemes of neutrino masses with experiments.

- Neutrino mass corresponds to a Lorentz invariant renormalizable term in the Lagrangian connecting a left ν_L and a right-handed field ν_R.
- Possible mass terms for neutral fermions can be written in two different ways. These are termed as Dirac and Majorana masses.

Neutrinos in SM.

In the Standard Model:

- there are no right-handed neutrinos (ν_R),
- there are only Higgs doublets of $SU(2)_L$,
- there are only renormalizable terms.

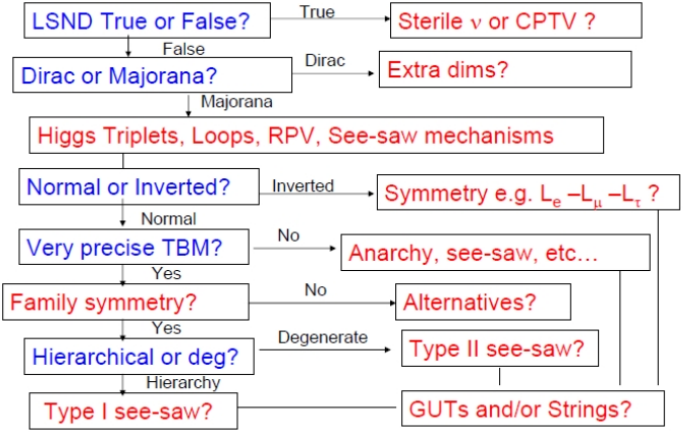
Experimental arguments.

- annomalus values of solar and atmospheric neutrino fluxes,
- LSND, KamLAND, K2K.

Theoretical predictions.

- all other fermions have mass,
- there are no symmetry principles forbiding neutrino mass terms for right handed neutrinos,

Mass Models open problems.



,,Neutrino Mass Models: a road map." S.F. King arXiv:0810.0492v1 [hep-ph]

Dirac Neutrinos.

Mass terms in SM.

All massive terms in SM occours in form:

$$\mathcal{L} = m\bar{\Psi}\Psi.$$

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Intorducing right-handed neutrinos ν_R into the SM:

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Intorducing right-handed neutrinos ν_R into the SM:

$$\mathcal{L}^D = -\left(\bar{\nu}_R m_D \nu_L + \bar{\nu}_L m_D \nu_R\right) + h.c. = m_D \bar{\nu}\nu.$$

where: m_D is in general 3×3 complex matrix.
we can generate neutrino mass from a coupling to the Higgs

$$\lambda_\nu < H > \bar{\nu}_L \nu_R \equiv m^\nu \bar{\nu}_L \nu_R,$$

where $< H >$ is Higgs vacuum expectation value.
Physical neutrino mass of $m^\nu \approx 0.2$ $[\mathrm{eV}]$ implies $\lambda_\nu \approx 10^{-12}$.

Majorana Neutrinos.

Majorana Neutrinos.

The form of a Majorana mass term is:

$$\mathcal{L}^M = -\frac{1}{2} m \left(\bar{\nu}_L \nu_L^C + \bar{\nu}_L^C \nu_L \right) = -\frac{1}{2} m \left(\bar{\nu}_L \mathbf{C} \nu_L^T + h.c. \right) = -\frac{1}{2} m \nu \bar{\nu}.$$

where $\nu = \nu_L + (\nu^C)_L$ is a self-conjugate two-component state satisfying $\nu = \nu^C = C\bar{\nu}^T$ where C is the charge conjugation matrix.

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where $\nu = \nu_L + (\nu^C)_L$ is a self-conjugate two-component state satisfying $\nu = \nu^C = C\bar{\nu}^T$ where C is the charge conjugation matrix. m must be generated by either an elementary Higgs triplet or by an effective operator involving two Higgs doublets arranged to transform as a triplet.

Dirac Majorana mass terms.

Dirac-Majorana

It is also possible to consider mixed models in which both Majorana and Dirac mass terms are present.

$$\mathcal{L}^{D-M} = -\frac{1}{2}\left(\bar{\nu}_L \bar{N}_L^C\right)\begin{pmatrix} m_T & m_D \\ m_D & m_S \end{pmatrix}\begin{pmatrix} \nu_R^C \\ N_R \end{pmatrix} + h.c.$$

m_T and m_S are Majorana masses which transform as weak triplets and singlets, respectively while m_D is a Dirac mass term.

Determination of M_ν.

Two groups of methods:

- *"top-down"* method:
  theoretical consideration of possible textures zeros and global symmetries which seems to arise from neutrino mass matrix structure.
  Analytical probes of determination M_ν in terms of neutrino masses values of neutrino mixing matrix elements.

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- *"top-down"* method:
  theoretical consideration of possible textures zeros and global symmetries which seems to arise from neutrino mass matrix structure.
  Analytical probes of determination M_ν in terms of neutrino masses values of neutrino mixing matrix elements.
- *"bottom-up"* method:
  relies on numerical analysis — diagonalization of many mass matrix textures leaving only these which are in agreement with present experimental data.

Texture Zeros In The Neutrino Mass Matrix.

One-zero textures of M_ν.

Pattern A	Pattern B	Pattern C
$\begin{pmatrix} \mathbf{0} & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}$	$\begin{pmatrix} \times & \mathbf{0} & \times \\ \mathbf{0} & \times & \times \\ \times & \times & \times \end{pmatrix}$	$\begin{pmatrix} \times & \times & \mathbf{0} \\ \times & \times & \times \\ \mathbf{0} & \times & \times \end{pmatrix}$
Pattern D	**Pattern E**	**Pattern F**
$\begin{pmatrix} \times & \times & \times \\ \times & \mathbf{0} & \times \\ \times & \times & \times \end{pmatrix}$	$\begin{pmatrix} \times & \times & \times \\ \times & \times & \mathbf{0} \\ \times & \mathbf{0} & \times \end{pmatrix}$	$\begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \mathbf{0} \end{pmatrix}$

Texture Zeros In The Neutrino Mass Matrix.

One-zero textures of M_ν implications.

Note that pattern A is of particular interest, because it predicts $\langle m \rangle_{ee} = 0$ (namely, the effective mass of the neutrinoless double beta decay vanishes).

$$m_{ee} = (Mu_\nu)_{ee} = (Mu_\nu)_{11} = \sum U_{ei}^2 m_i.$$

While:

- $\langle m \rangle_{ee} \neq 0$ must imply that neutrinos are Majorana particles,

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$$m_{ee} = (Mu_\nu)_{ee} = (Mu_\nu)_{11} = \sum U_{ei}^2 m_i .$$

While:

- $\langle m \rangle_{ee} \neq 0$ must imply that neutrinos are Majorana particles,
- $\langle m \rangle_{ee} = 0$ does not *necessarily* imply that neutrinos are Dirac particles.

TBM.

The lepton mixing determined from the results of neutrino experiments can be well described by the so called Tri-Bimaximal Mixing (TBM) matrix:

$$U_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

$$\sin^2\theta_{23} = \frac{1}{2}, \quad \sin\theta_{13} = 0, \quad \sin^2\theta_{12} = \frac{1}{3}.$$

In terms of the standard parameterization of lepton mixing matrix:

$$U_{PMNS} = U_{23}(\theta_{23})\Gamma_\delta U_{13}(\theta_{13})\Gamma_\delta^\star U_{12}(\theta_1 2).$$

TBM.

For the Majorana neutrinos in the flavor basis $(\nu_e, \nu_\mu, \nu_\tau)$ the mass matrix which leads to the TBM mixing equals:

$$m_{TBM} = U_{TBM} m_\nu^{diag} U_{TBM}^T$$

In general, m_i are complex and we can represent them as:

$$m_1 = |m_1|, \quad m_2 = |m_2| e^{i2\phi_2}, \quad m_3 = |m_3| e^{i2\phi_3}$$

Here ϕ_1 and ϕ_2 are the Majorana CP-violating phases.

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Here ϕ_1 and ϕ_2 are the Majorana CP-violating phases. We can find explicitly:

$$m_{TBM} = \begin{pmatrix} a & b & c \\ \dots & \frac{1}{2}(a+b+c) & \frac{1}{2}(a+b-c) \\ \dots & \dots & \frac{1}{2}(a+b+c) \end{pmatrix}$$

Large Neutrino Mixings.

Bimaximal.

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad m_{BM} = \begin{pmatrix} a+b & c & c \\ c & a & b \\ c & b & a \end{pmatrix}$$

Democratic.

$$U_{D} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \quad m_{D} = \begin{pmatrix} a & b & \sqrt{2}b \\ b & a-2c & \sqrt{2}c \\ \sqrt{2}b & \sqrt{2}c & a-c \end{pmatrix}$$

Texture Zeros In The Neutrino Mass Matrix.

Two-zero textures of M_ν.

There are totally fifteen $\left(\frac{6!}{n!(6-n)!}\right)$ possible patterns of M_ν with two independent vanishing entries. But only seven of them are found to be compatible with current neutrino oscillation data:

Pattern A_1	Pattern A_2	Pattern B_1
$\begin{pmatrix} \mathbf{0} & \mathbf{0} & \times \\ \mathbf{0} & \times & \times \\ \times & \times & \times \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & \times & \mathbf{0} \\ \times & \times & \times \\ \mathbf{0} & \times & \times \end{pmatrix}$	$\begin{pmatrix} \times & \times & \mathbf{0} \\ \times & \mathbf{0} & \times \\ \mathbf{0} & \times & \times \end{pmatrix}$
Pattern B_2	Pattern B_3	Pattern B_4
$\begin{pmatrix} \times & \mathbf{0} & \times \\ \mathbf{0} & \times & \times \\ \times & \times & \mathbf{0} \end{pmatrix}$	$\begin{pmatrix} \times & \mathbf{0} & \times \\ \mathbf{0} & \mathbf{0} & \times \\ \times & \times & \times \end{pmatrix}$	$\begin{pmatrix} \times & \times & \mathbf{0} \\ \times & \times & \times \\ \mathbf{0} & \times & \mathbf{0} \end{pmatrix}$
Pattern C		
$\begin{pmatrix} \times & \times & \times \\ \times & \mathbf{0} & \times \\ \times & \times & \mathbf{0} \end{pmatrix}$		

Texture Zeros In The Neutrino Mass Matrix.

Three-zero textures of M_ν implications.

There are twenty three-zero patterns of M_ν which can be classified into four categories:

- Type 0 with all three diagonal matrix elements vanishing:

$$M_0 = \begin{pmatrix} \mathbf{0} & \times & \times \\ \times & \mathbf{0} & \times \\ \times & \times & \mathbf{0} \end{pmatrix},$$

- Type I with two diagonal matrix elements vanishing:

$$M_{\mathrm{I}_1} = \begin{pmatrix} \mathbf{0} & \times & \mathbf{0} \\ \times & \mathbf{0} & \times \\ \mathbf{0} & \times & \times \end{pmatrix}, \quad M_{\mathrm{I}_7} = \begin{pmatrix} \mathbf{0} & \mathbf{0} & \times \\ \mathbf{0} & \mathbf{0} & \times \\ \times & \times & \times \end{pmatrix}.$$

Texture Zeros In The Neutrino Mass Matrix.

Three-zero textures of M_ν implications.

There are twenty three-zero patterns of M_ν which can be classified into four categories:

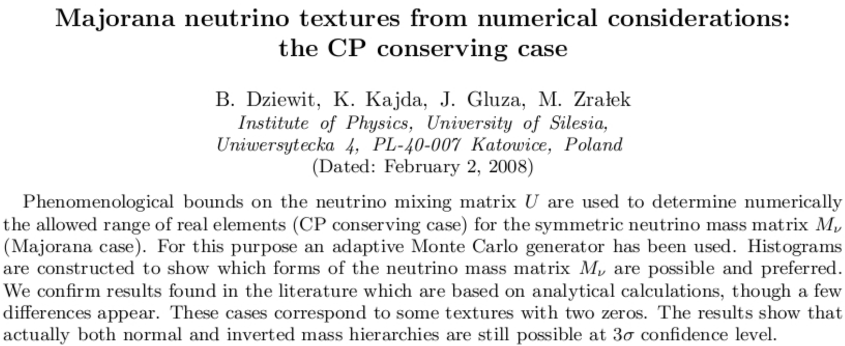
- Type II with one diagonal matrix element vanishing:

$$M_{\mathrm{II}_1} = \begin{pmatrix} \times & \times & \mathbf{0} \\ \times & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \times \end{pmatrix}, \quad M_{\mathrm{II}_7} = \begin{pmatrix} \times & \times & \mathbf{0} \\ \times & \times & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}.$$

- Type III with three diagonal matrix elements non-vanishing:

$$M_{\mathrm{III}} = \begin{pmatrix} \times & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \times & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \times \end{pmatrix}.$$

Real Majorana Mass Matrix.



Majorana neutrino textures from numerical considerations: the CP conserving case

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(Dated: February 2, 2008)

Phenomenological bounds on the neutrino mixing matrix U are used to determine numerically the allowed range of real elements (CP conserving case) for the symmetric neutrino mass matrix M_ν (Majorana case). For this purpose an adaptive Monte Carlo generator has been used. Histograms are constructed to show which forms of the neutrino mass matrix M_ν are possible and preferred. We confirm results found in the literature which are based on analytical calculations, though a few differences appear. These cases correspond to some textures with two zeros. The results show that actually both normal and inverted mass hierarchies are still possible at 3σ confidence level.

Phys. Rev. D 74 (2006) 033003 [arXiv:hep-ph/0604193].

Real Majorana Mass Matrix.

In this paper the real symmetric 3×3 neutrino mass matrix is analyzed. It means that we assume directly the Majorana nature of neutrinos and that the investigation is restricted to the CP conserving case. A general neutrino mass matrix M_ν which we analyze has the following form:

$$\begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix} \tag{1}$$

Real Majorana Mass Matrix.

In this paper the real symmetric 3×3 neutrino mass matrix is analyzed. It means that we assume directly the Majorana nature of neutrinos and that the investigation is restricted to the CP conserving case. A general neutrino mass matrix M_ν which we analyze has the following form:

$$\begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix} \tag{1}$$

The standard neutrino theory involves diagonalization of the neutrino mass matrix M_ν by use of the mixing matrix U:

$$m_{diag} = U^T M_\nu U. \tag{2}$$

Real Majorana Mass Matrix.

$$m_i \leq \ 2.2\,\mathrm{eV}, \quad |m_i - m_j| < 0.05\,\mathrm{eV}, \quad i,j = 1,2,3. \qquad (3)$$

i	x_i	x_i^{cent}	σ_i
1	Δm^2_{32}	$2.6 \cdot 10^{-3}$	10^{-3}
2	Δm^2_{21}	$8.3 \cdot 10^{-5}$	10^{-5}
3	$\|U_{e1}\|$	0.835	0.045
4	$\|U_{e2}\|$	0.54	0.07
5	$\|U_{e3}\|$	0.1	0.1
6	$\|U_{\mu 1}\|$	0.355	0.165
7	$\|U_{\mu 2}\|$	0.575	0.155
8	$\|U_{\mu 3}\|$	0.7	0.12
9	$\|U_{\tau 1}\|$	0.365	0.165
10	$\|U_{\tau 2}\|$	0.59	0.15
11	$\|U_{\tau 3}\|$	0.685	0.125

The allowed absolute values of the neutrino mass squared differences Δm^2_{32}, Δm^2_{21} and the allowed absolute values of the neutrino mixing matrix elements $|U_{ij}|$. x_i^{cent} and σ_i are the central values and the 3σ uncertainties, respectively.

Real Majorana Mass Matrix.

First step — Scattering.

- *Random generation of input parameters.*

Real Majorana Mass Matrix.

First step — Scattering.

- *Random generation of input parameters.*
- *Diagonalization of the neutrino mass matrix M_ν.*

Real Majorana Mass Matrix.

First step — Scattering.

- *Random generation of input parameters.*
- *Diagonalization of the neutrino mass matrix M_ν.*
- *Comparison with experimental results and saving allowed parameters*

$$\chi_i^2 = \frac{(x_i^{cent} - x_i)^2}{\left(\frac{\sigma_i}{\alpha}\right)^2}.$$

Real Majorana Mass Matrix.

Second Step — The Adaptive Monte Carlo.

- *Reading obtained points*
  (a) *Random generation of input parameters*

$$x_i^{cent} \pm \xi_{it}\delta_i, \qquad \xi_{it} = \begin{cases} 1 & it = 0, \\ 0.6/it & it > 0. \end{cases}$$

.

  (b) *Diagonalization*
  (c) *Comparison with experimental data and saving successive cases*

Real Majorana Mass Matrix.

Second Step — The Adaptive Monte Carlo.

- *Reading obtained points*
  (a) *Random generation of input parameters*

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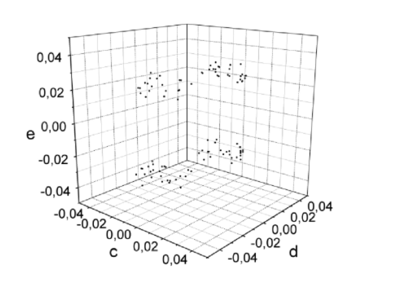
.

  (b) *Diagonalization*
  (c) *Comparison with experimental data and saving successive cases*
- *Setting new central values.*

$$\chi^2 = \sum_{i=1}^{11} \chi_i^2.$$

Real Majorana Mass Matrix - results.

General case.



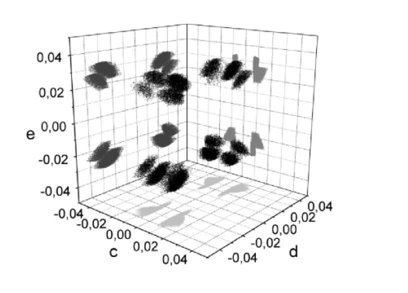


Figure: Three dimensional plots of allowed parameters found by the AMC procedure. On the left plot there are points obtained firstly by generating random parameters, on the right plot the points are denser as AMC looks for additional solutions in a vicinity of the parameters obtained in the first step.

Real Majorana Mass Matrix - results.

General case — normal hierarchy.

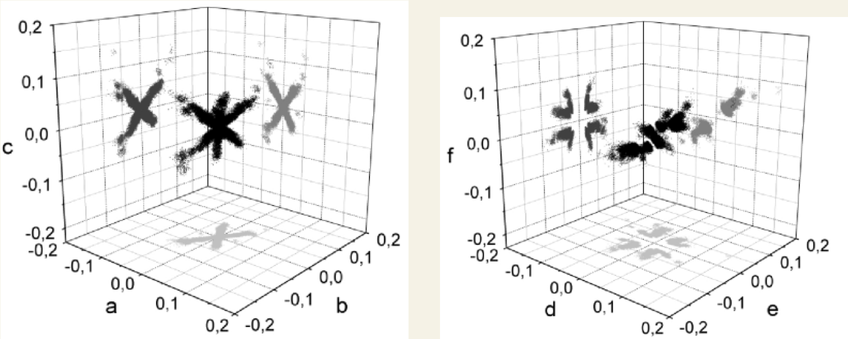


Figure: Allowed regions of parameters for the neutrino mass matrix M_ν with present experimental data (3σ level), the general case with normal mass hierarchy.

Real Majorana Mass Matrix - results

General case — normal hierarchy.

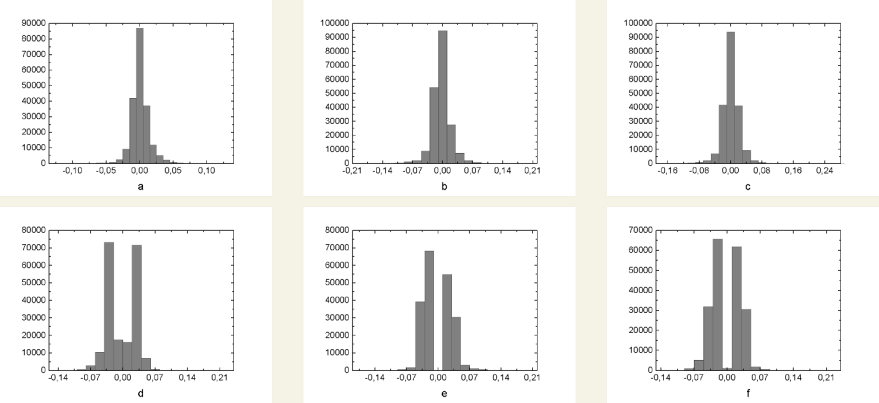
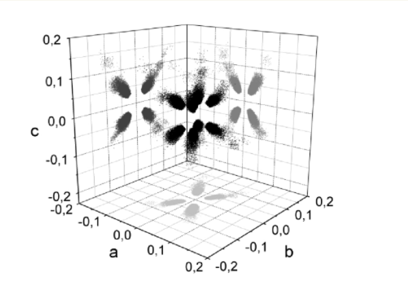


Figure: Frequency spectrum for the elements of the neutrino mass matrix M_ν: the general case with normal mass hierarchy.

Real Majorana Mass Matrix - results

General case - inverted hierarchy.



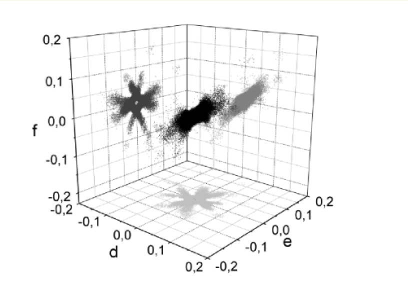


Figure: Allowed regions of parameters for the neutrino mass matrix M_ν with present experimental data (3σ level), the general case with inverted mass hierarchy.

Real Majorana Mass Matrix - results

General case — inverted hierarchy.

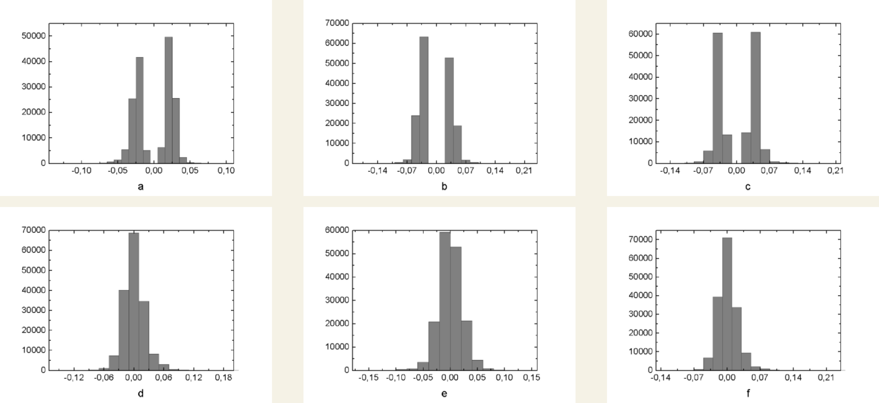


Figure: Frequency spectrum for the elements of the neutrino mass matrix M_ν: the general case with inverted mass hierarchy.

Real Majorana Mass Matrix - results

One texture zero — A:

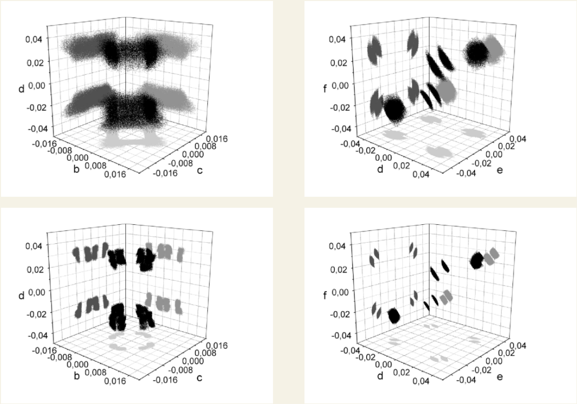


Figure: Allowed regions for the mass matrix with $a = 0$ (A texture). The first row shows plots with $\alpha = 1$ (present data, 3σ level), the second row shows results for $\alpha = 2$.

Real Majorana Mass Matrix - results

One texture zero — A:

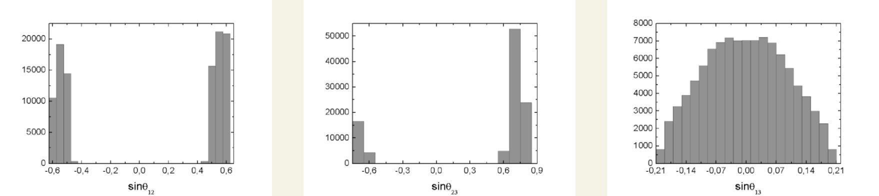


Figure: Histograms of neutrino rotation angles $\sin\theta_{12}$, $\sin\theta_{23}$ and $\sin\theta_{13}$ for neutrino mass texture with one zero $a = 0$. The histogram for $\sin\theta_{23}$ does not depend on a and is the same for $a \neq 0$.

Real Majorana Mass Matrix - results

One texture zero — A:

TEXTURE	ZERO PARAMETERS	MASS RANGE	MASS MEAN	α_0
A normal	$a = 0$	$\mid m_3 \mid = (0.041, 0.062)$ $\mid m_2 \mid = (0.009, 0.015)$ $\mid m_1 \mid = (0.002, 0.011)$ $\mid m_{\beta\beta0\nu} \mid = 0$	$< m_3 >= 0.052$ $< m_2 >= 0.010$ $< m_1 >= 0.005$ $< m_{\beta\beta0\nu} >= 0$	3.32

Table: Masses and effective neutrinoless double beta decay mass parameter $< m_{\beta\beta0\nu} >$ for allowed textures with one with one zero. The last column shows the parameter α_0 for which schemes have no positive solutions.

Real Majorana Mass Matrix - results

Two texture zeros:

TEXTURE	*ZERO PARAMETERS*	*MASS RANGE*	*MASS MEAN*	α_0
A_1 and A_2 normal	$a, b = 0$ $a, c = 0$	$\mid m_3 \mid = (0.041, 0.062)$ $\mid m_2 \mid = (0.009, 0.015)$ $\mid m_1 \mid = (0.002, 0.012)$ $\mid m_{\beta\beta0\nu} \mid = 0$	$< m_3 >= 0.053$ $< m_2 >= 0.011$ $< m_1 >= 0.004$ $< m_{\beta\beta0\nu} >= 0$	2.65
B_1, B_2 degenerate or normal	$c, d = 0$ $b, f = 0$	$\mid m_3 \mid = (0.05, 0.14)$ $\mid m_2 \mid = (0.03, 0.13)$ $\mid m_1 \mid = (0.02, 0.13)$ $\mid m_{\beta\beta0\nu} \mid = (0.02, 0.13)$	$< m_3 >= 0.08$ $< m_2 >= 0.06$ $< m_1 >= 0.06$ $< m_{\beta\beta0\nu} >= 0.06$	1.18
B_1, B_2 degenerate or inverted	$c, d = 0$ $b, f = 0$	$\mid m_2 \mid = (0.05, 0.18)$ $\mid m_1 \mid = (0.05, 0.18)$ $\mid m_3 \mid = (0.03, 0.17)$ $\mid m_{\beta\beta0\nu} \mid = (0.05, 0.18)$	$< m_2 >= 0.09$ $< m_1 >= 0.09$ $< m_3 >= 0.07$ $< m_{\beta\beta0\nu} >= 0.09$	1.18
B_3, B_4 degenerate or normal	$b, d = 0$ $c, f = 0$	$\mid m_3 \mid = (0.05, 0.22)$ $\mid m_2 \mid = (0.025, 0.21)$ $\mid m_1 \mid = (0.02, 0.205)$ $\mid m_{\beta\beta0\nu} \mid = (0.03, 0.21)$	$< m_3 >= 0.08$ $< m_2 >= 0.06$ $< m_1 >= 0.06$ $< m_{\beta\beta0\nu} >= 0.06$	1.25
B_3, B_4 degenerate or inverted	$b, d = 0$ $c, f = 0$	$\mid m_2 \mid = (0.05, 0.25)$ $\mid m_1 \mid = (0.045, 0.25)$ $\mid m_3 \mid = (0.03, 0.24)$ $\mid m_{\beta\beta0\nu} \mid = (0.045, 0.246)$	$< m_2 >= 0.083$ $< m_1 >= 0.082$ $< m_3 >= 0.065$ $< m_{\beta\beta0\nu} >= 0.084$	1.25
C inverted	$d, f = 0$	$\mid m_2 \mid = (0.042, 0.072)$ $\mid m_1 \mid = (0.041, 0.071)$ $\mid m_3 \mid = (0.012, 0.039)$ $\mid m_{\beta\beta0\nu} \mid = (0.011, 0.039)$	$< m_2 >= 0.056$ $< m_1 >= 0.055$ $< m_3 >= 0.023$ $< m_{\beta\beta0\nu} >= 0.022$	2.65

Real Majorana Mass Matrix - results.

Two texture zeros.

i	x_i	x_i^{cent}	σ_i	A_1, A_2	$B_1 - B_4$	C
1	Δm^2_{32}	$2.6 \cdot 10^{-3}$	10^{-3}	$2.65 \cdot 10^{-3}$	$2.55 \cdot 10^{-3}$	$2.61 \cdot 10^{-3}$
2	Δm^2_{21}	$8.3 \cdot 10^{-5}$	10^{-5}	$8.27 \cdot 10^{-5}$	$8.35 \cdot 10^{-5}$	$8.30 \cdot 10^{-5}$
3	$\lvert U_{e1} \rvert$	0.835	0.045	0.84	0.84	0.84
4	$\lvert U_{e2} \rvert$	0.54	0.07	0.54	0.54	0.54
5	$\lvert U_{e3} \rvert$	0.1	0.1	0.12	$1.9 \cdot 10^{-3}$	0.06
6	$\lvert U_{\mu 1} \rvert$	0.355	0.165	0.41	0.40	0.36
7	$\lvert U_{\mu 2} \rvert$	0.575	0.155	0.56	0.63	0.58
8	$\lvert U_{\mu 3} \rvert$	0.7	0.12	0.72	0.66	0.72
9	$\lvert U_{\tau 1} \rvert$	0.365	0.165	0.36	0.36	0.41
10	$\lvert U_{\tau 2} \rvert$	0.59	0.15	0.63	0.55	0.59
11	$\lvert U_{\tau 3} \rvert$	0.685	0.125	0.68	0.75	0.68

Table: This table shows x_i^{cent} values obtained from numerical solutions for two zero textures. It appears, that cases A_1 and A_2 coincide with solutions for one zero texture with $a = 0$.

Real Majorana Mass Matrix - results

Two texture zeros — C:

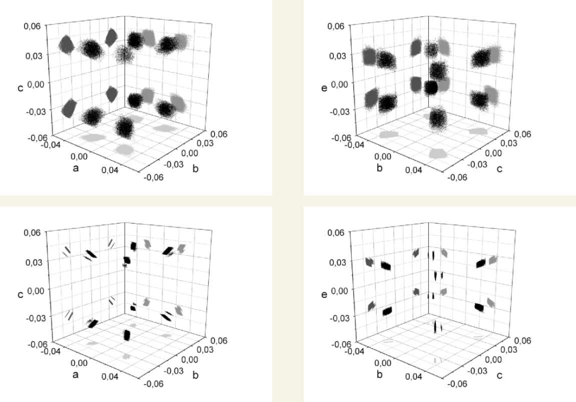


Figure: Allowed regions for the mass matrix with $d, f = 0$ (C texture). The first row shows plots with $\alpha = 1$ (present data, 3σ level). The second row shows results for $\alpha = 2$.

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- textures B give small values of $\sin\theta_{13}$.
- cases with $m_{\beta\beta0\nu} = 0$ have got only normal mass hierarchy and they all imply similar results.

Complex Neutrino Mass Matrix.

$$\mathcal{M}_\nu = W \cdot m_{diag} \cdot W^\star, \quad W = f \cdot U^\star_{PMNS} \cdot P,$$

where:

$$f = \begin{pmatrix} e^{-\imath\beta_1} & 0 & 0 \\ 0 & e^{-\imath\beta_2} & 0 \\ 0 & 0 & e^{-\imath\beta_3} \end{pmatrix}, \; P = \begin{pmatrix} e^{-\imath\alpha_1} & 0 & 0 \\ 0 & e^{-\imath\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

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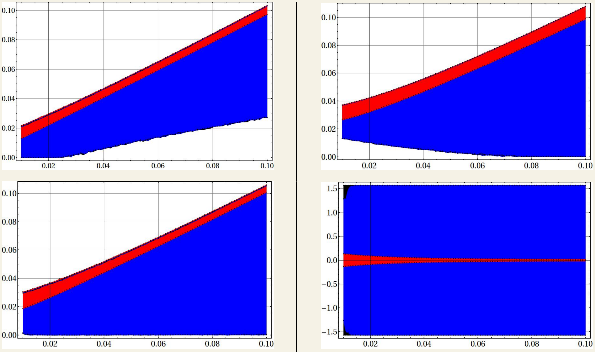
We can express all terms (separatly Im and Re parts) of $\mathcal{M}_\nu$ as a function of:

$$\theta_{12}, \theta_{13}, \theta_{23}, m_1, m_2, m_3, \delta, \alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3.$$

and found their minimal and maximal values for current experimental data.

Complex Neutrino Mass Matrix.



$$\begin{pmatrix} |a|e^{i\varphi_a} & |b|e^{i\varphi_b} & |c|e^{i\varphi_c} \\ |b|e^{i\varphi_b} & |d|e^{i\varphi_d} & |e|e^{i\varphi_e} \\ |c|e^{i\varphi_c} & |e|e^{i\varphi_e} & |b|e^{i\varphi_f} \end{pmatrix} \qquad \begin{pmatrix} |a|e^{i\varphi_a} & |b|e^{i\varphi_b} & |c|e^{i\varphi_c} \\ |b|e^{i\varphi_b} & |d|e^{i\varphi_d} & |e|e^{i\varphi_e} \\ |c|e^{i\varphi_c} & |e|e^{i\varphi_e} & |f|e^{i\varphi_f} \end{pmatrix}$$

Figure: Red scope — $|M_{ij}|, \varphi_{ij}$ with: $\delta \neq, \alpha_i = 0, \beta_i = 0$, blue with: $\delta \neq, \alpha_i \neq 0, \beta_i = 0$.

Complex Neutrino Mass Matrix.

Recent work and future plans.

- non CP conserving case for real neutrino mass matrix,
- complex neutrino mass matrix:
  - histograms and correspondence to real case ,
  - program for automatic distinguish between possible textures,
- more general cases . . . 6×6 neutrino mass matrix . . .