Martini-Marteau and Nieves-Oset models in neutrino scattering

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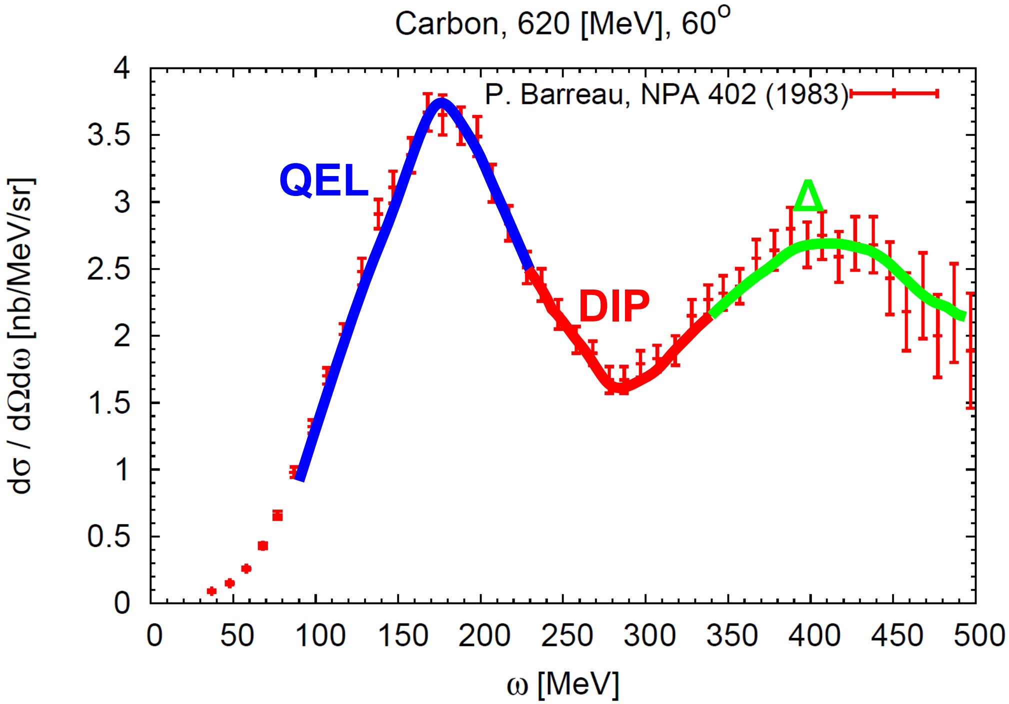
Motivation

- Neutrino interactions at energy transfers below 1 [GeV]: modern experiments (MiniBooNe, T2K...)
- Enough energy to reach multi-particle emissions ($N,\ \pi$, next slide), need for a consistent model.
- Problems with detection of multiparticle final states, easy confusion with QEL.
- No new physics without the understanding of ν-nucleus process.

Motivation- nuclear dynamics



- QEL: mainly $1p1h$ excitation, some contribution from $npnh$?
- Δ: mainly excitation of the Δ resonance, (mainly) $1p1h1\pi$ production, but $npnh$ possible!
- DIP: QEL and Δ tails, Meson Exchange Currents, a lot of $2p2h$.

Common part: polarisation tensor

- Two main groups: M. Martini *et.al.* and J. Nieves *et.al.*
- Typical inclusive cross-section formula ($q^\mu = (\omega, \boldsymbol{q})$):

$$d\sigma \propto L_{\mu\nu} W^{\mu\nu}$$

$$W^{\mu\nu} = \overline{\sum_i} \sum_f \Omega E_i \int d^3x e^{-i(\boldsymbol{q}+\boldsymbol{p_i}-\boldsymbol{p_f})\boldsymbol{x}} \left\langle i \left| \hat{J}^{\dagger\nu}(0) \right| f \right\rangle \left\langle f \left| \hat{J}^{\mu}(0) \right| i \right\rangle \delta(E_f - E_i - \omega)$$

- $\hat{J}^\mu \to$ 1,2...n- body nuclear currents. Most simple form: truncated to 1-body vector+axial current.

$$\left\langle p', s' \left| \hat{J}^\mu \right| p, s \right\rangle = \overline{u}_{s'}(\boldsymbol{p'}) \left[F_1(q)\gamma^\mu + i\sigma^{\mu\alpha} q_\alpha \frac{F_2(q)}{2M} + G_A(q)\gamma^5 + G_P(q)\gamma^5 \frac{q^\mu}{2M} \right] u_s(\boldsymbol{p})$$

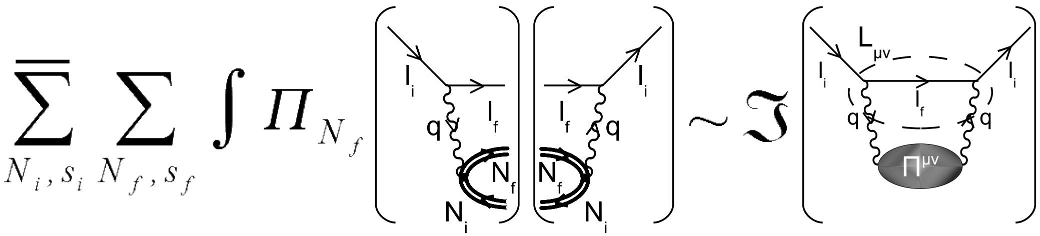
- From general q.m. and complex analysis properties :

$$\left(\delta(x) = -\frac{1}{\pi} \Im \frac{1}{x + i\epsilon}, \ \sum_f |f\rangle\langle f| = \mathbf{1} \ldots\right)$$

$$L_{\mu\nu} W^{\mu\nu} = -\frac{1}{\pi} \Im (L_{\mu\nu} \Pi^{\mu\nu})$$

Common part: polarisation tensor

- The ideology of polarisation tensor:



$$
\begin{aligned}
\Pi^{\mu\nu}(q) &= \Omega M_T \int d^4x e^{iqx} \left\langle i \left| T\left\{ J^{\nu\dagger}(x) J^{\mu}(0) \right\} \right| i \right\rangle = \\
&= \Omega M_T \int d^4x e^{iqx} \left\langle i \left| T\left\{ J_I^{\nu\dagger}(x) J_I^{\mu}(0) exp^{i \int d^4x \mathcal{L}_{int}(x)} \right\} \right| i \right\rangle
\end{aligned}
$$

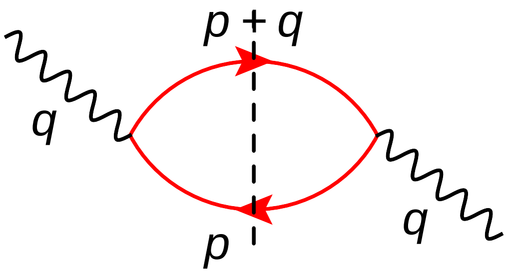
- Perturbation expansions and Feynman graph level analysis possible
- Polarisation tensor: gauge boson self-energy in nuclear matter.

Common part: polarisation tensor

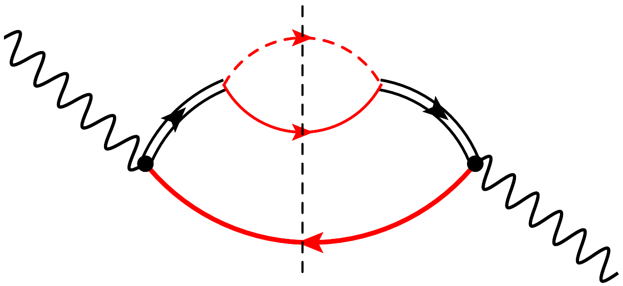
- Imaginary part of $\Pi^{\mu\nu} \to$ final state particle propagators on-shell:

$$\Im G(p) \propto \Im \frac{1}{p^2 - M^2 + i\epsilon} = -2\pi\delta(p^2 - M^2)\Theta(p^0)$$

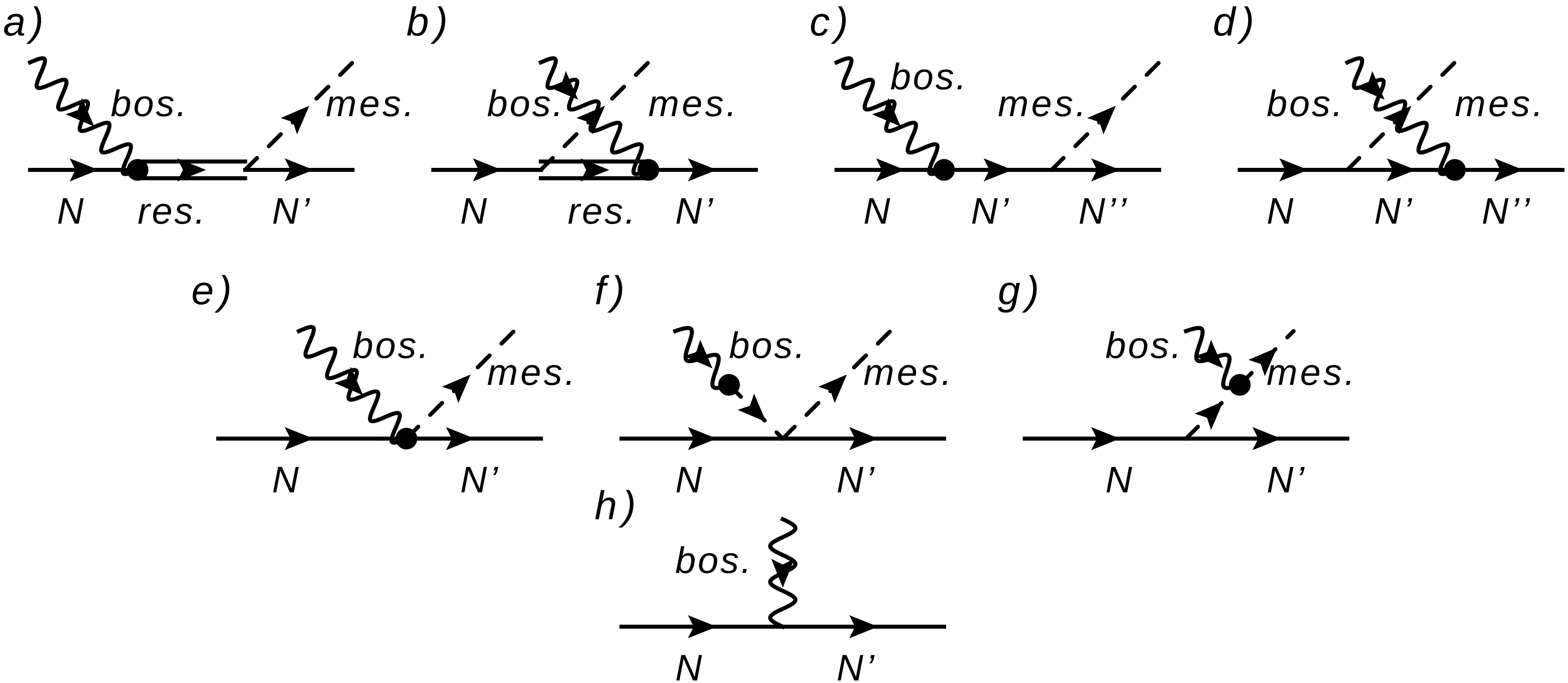
- Example: Fermi Gas



- Example: Resonant π production

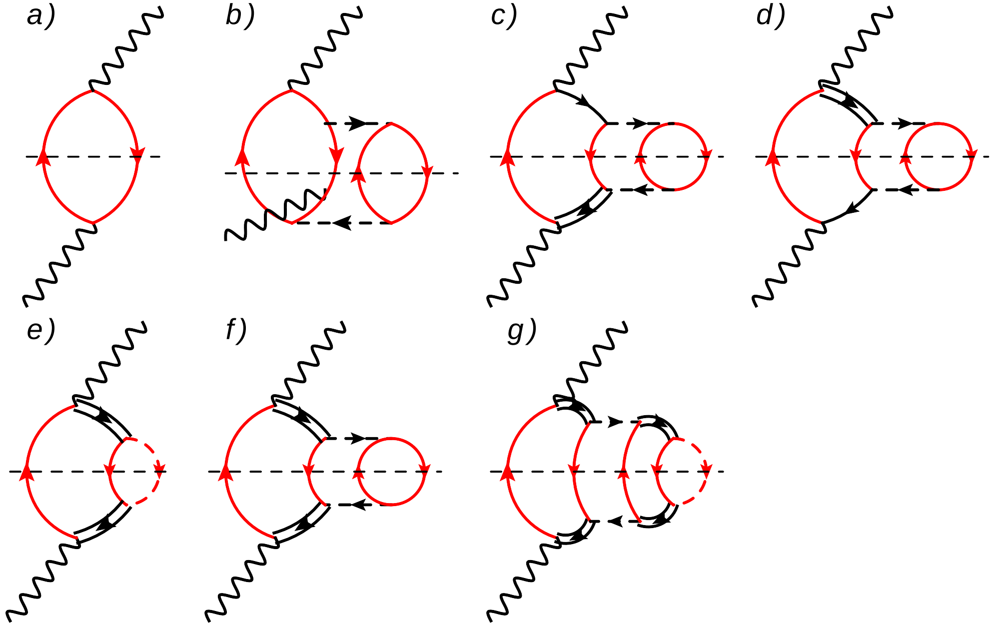


Basic elements



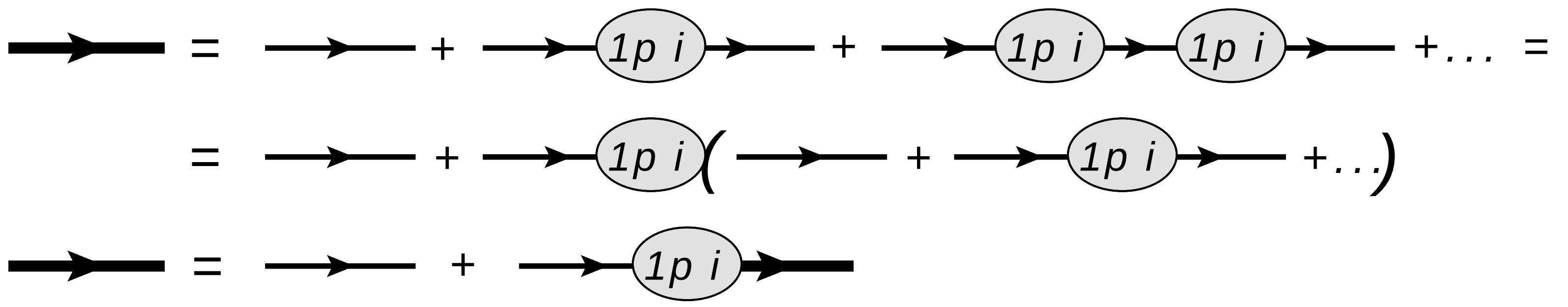
- *bos.* $\rightarrow$ $W^{\pm}, Z^{0}, \gamma$, *mes.* $\rightarrow$ $\pi^{\pm}, \pi^{0}, \rho^{\pm}_{\mu}, \rho^{0}_{\mu} \ldots$
  res. $\rightarrow$ resonances (here- Δ).

Martini-Marteu (PRC 80,2009)



- Graphs $e, f, g:$ Δ self-energy in nuclear matter, parametrisation of Oset NPA468
- $2p2h$ parametrised after Delorme and Guichon
- Apparent lack of most MEC
- Usage of LDA

△ Self-energy model



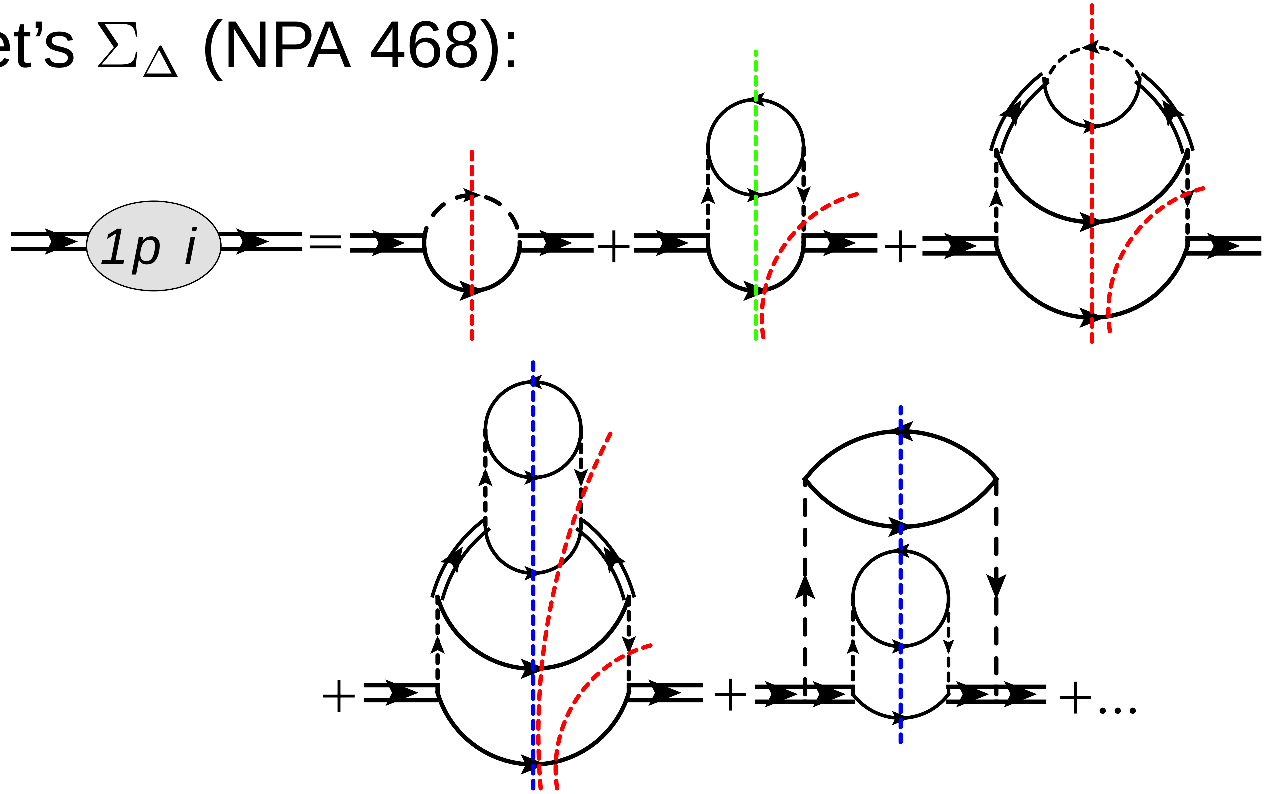
- Dyson equation as a sum of one-particle-irreducible insertions. Modification of free propagator:

$$G(\boldsymbol{p},\omega) = \frac{1}{[G^0(\boldsymbol{p},\omega)]^{-1} - \Sigma(\boldsymbol{p},\omega)}$$

- Usually matrix equations.

$\triangle$ Self-energy model

- Oset's Σ_Δ (NPA 468):



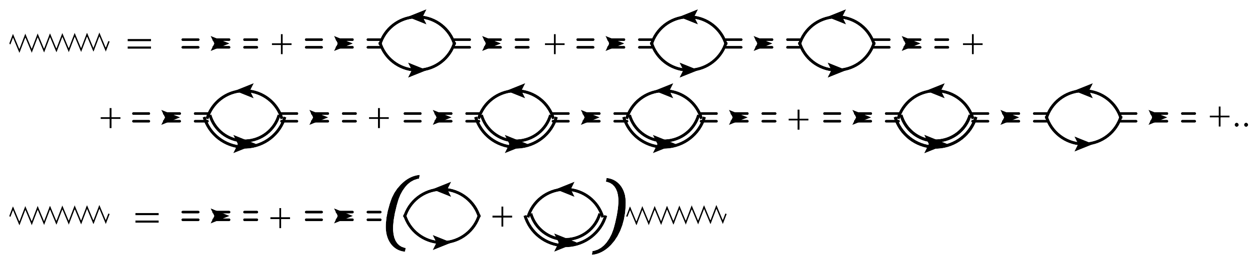
- Physical channels:
  $Im\Sigma_\Delta = \Im\Sigma_{QEL} + \frac{1}{2}\tilde{\Gamma} + \Im\Sigma_{A2} + \Im\Sigma_{A3}$. $\frac{1}{2}\tilde{\Gamma}$ - $\Delta \to \pi N$ width with inclusion of Pauli blocking (first graph). More, than Martini claims, but Marteu PHD shows them all.

$\triangle$ Self-energy model

- Oset: replacement of π line with effective $\rho + \pi +$ Landau-Migdal repulsion

$$V_{ij}(q) = V_l(q)\hat{\boldsymbol{q}}_i\hat{\boldsymbol{q}}_j + V_t(q)(1 - \hat{\boldsymbol{q}}_i\hat{\boldsymbol{q}}_j)$$

- Renormalisation by the ph and Δh excitations:



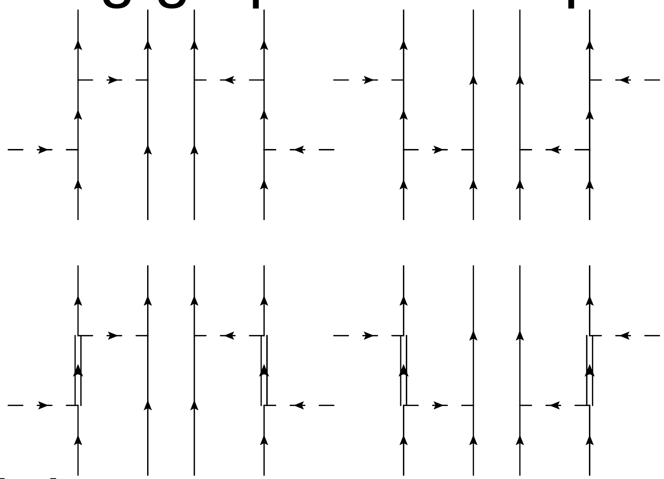
- Replacement of the bare interaction with induced one $W(q)$:

$$W(q) = \frac{V(q)}{1 - V(q)(U_N(q) + U_\Delta(q))};\ U \to Lindhard\ functions$$

- Cuts: ρ on shell? $m_\rho \approx 770[MeV]$, well outside limits of a nonrelativistic model!

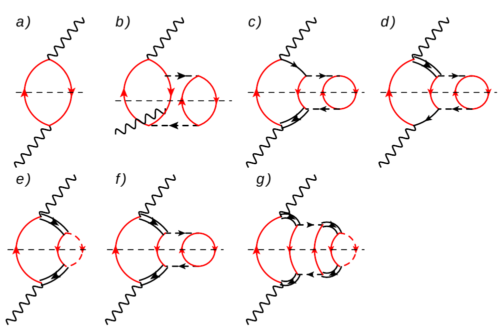
The $2p2h$ parametrisation

- Guichon-Delorme: extrapolation to $W^{\pm}$ of the 2N pion absorption model of Shimizu-Faessler **NPA 433.**
- Following graphs: absorptive part of optical potential.



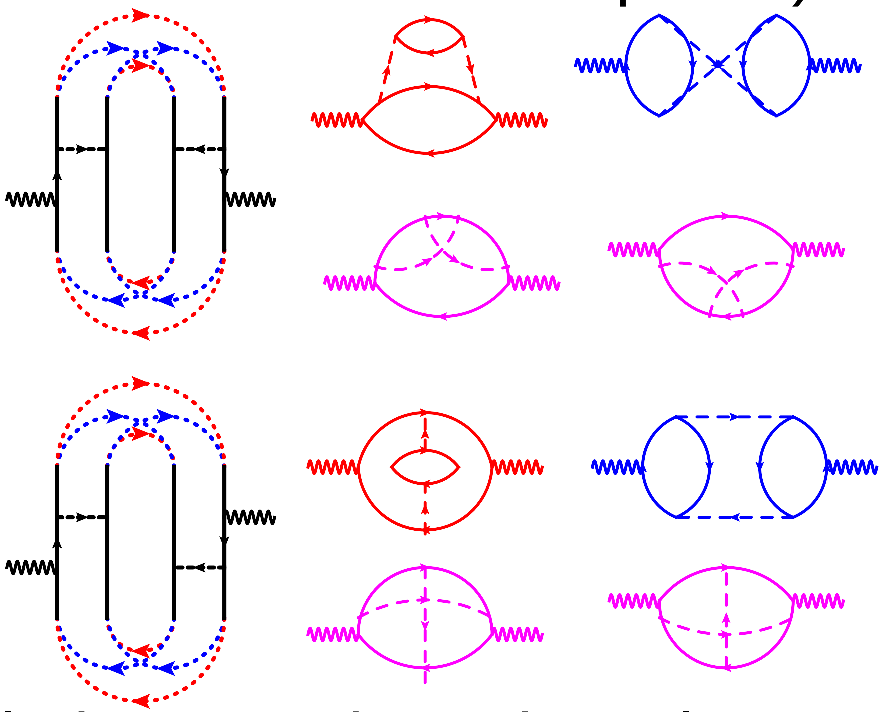
$$\Rightarrow Im(V_{opt}(q))$$

- Martini:



Is that it?

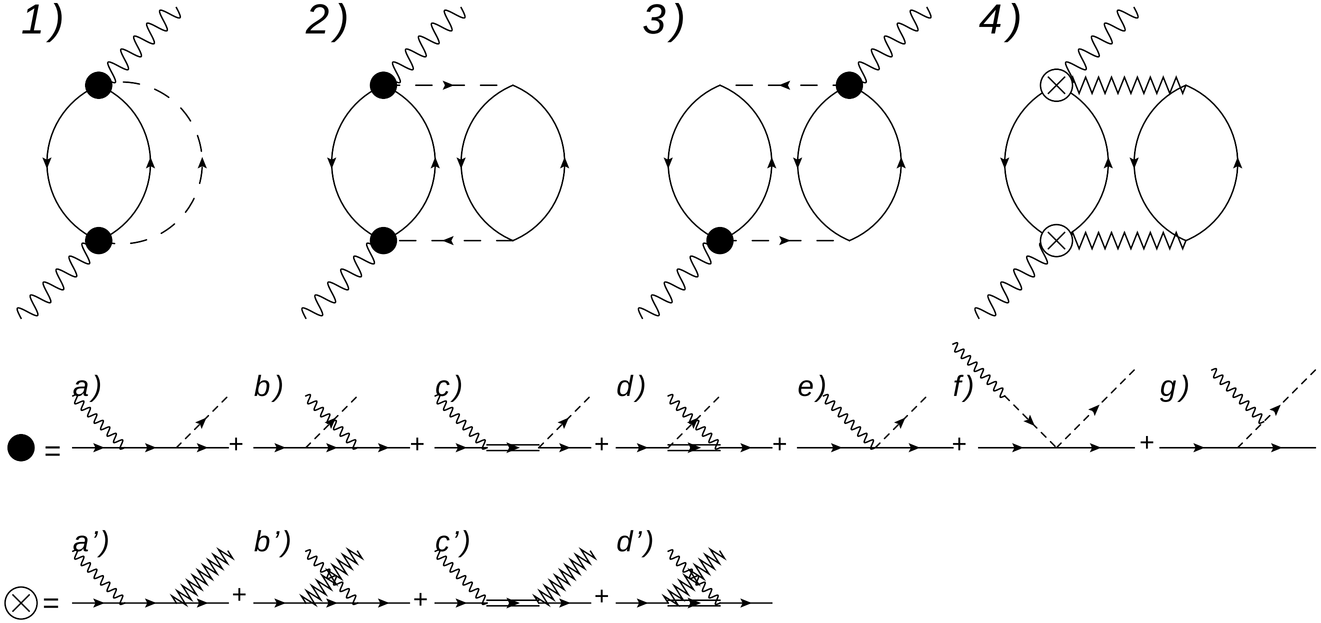
- Possible polarisation propagators of some Shimizu graphs (red: two-nucleon absorptions):



- Martini includes $2p2h$ through nucleon self-energy too! (He admits that in PRC 81, 2010!)
- Blue graphs- MEC+ MEC corr (π crossed)!, Violet: nucleon self-energy+ $p-h$ interaction. For all possibilities add$\Delta - \Delta$ and $\Delta - N$ and $N - \Delta$.

Nieves basic polarisation

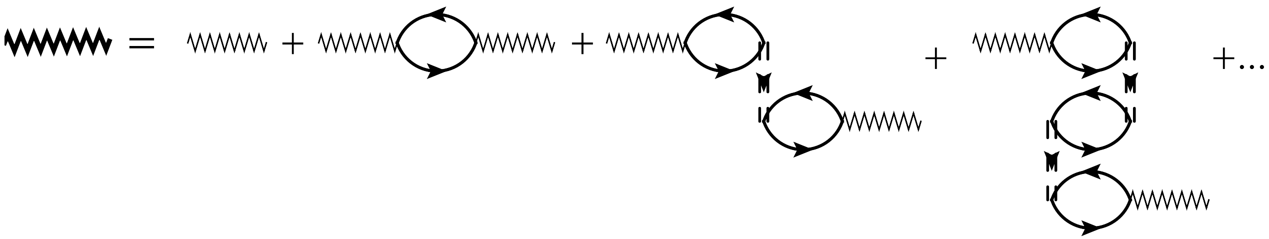
- Basic polarisation tensor components by Nieves (arXiv:1102.2777v1 [hep-ph]):



- Single pion production ($1)$), $2p2h$ through π ($2)$), extra MEC ($3)$), $2p2h$ through ρ ($4)$) Graphs $e),\ f),\ g)$ and ρ-meson exchange $a'),\ b'),\ c')$ and their interferences absent in Martini-Marteau (Contact term $e)$ and some of the MEC in Marteau's PhD! What happened?).

Nieves improvements:

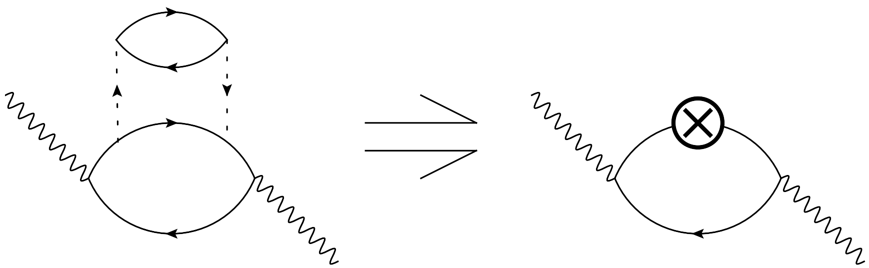
- Usage of LDA with nucleon energy $E(\boldsymbol{p}) = \sqrt{\boldsymbol{p}^2 + M^2} - \frac{k_F^2}{2M}$, with local Fermi momentum $k_F(r) = (3\pi^2\rho(r)/2)^{1/3}$.
- Replacement of free π and ρ propagators with their ph + Δh pion selfenergy

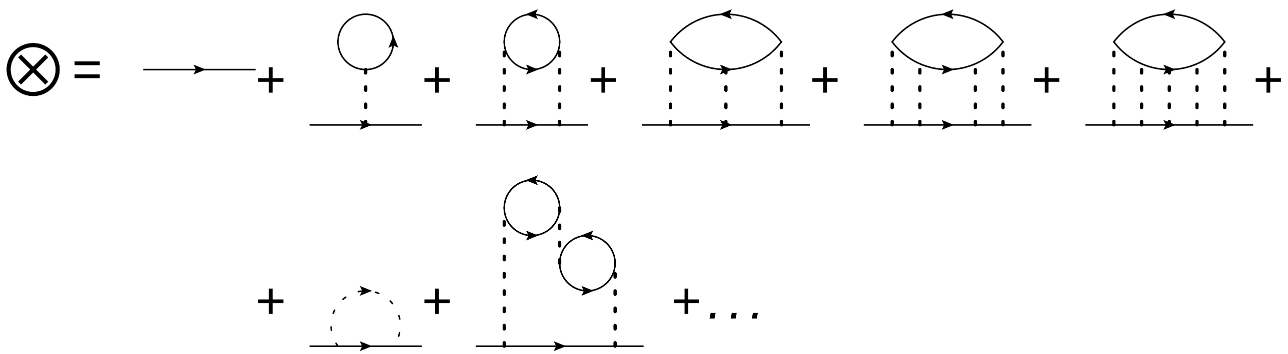


double-dashed line: effective interaction, explicit π and ρ exchange between forward and backward going bubbles+ repulsive L-M parameter g'.

Nieves improvements:

- Replacement of following $2p2h$ graph with nucleon selfenergy (PRC 46,1992):

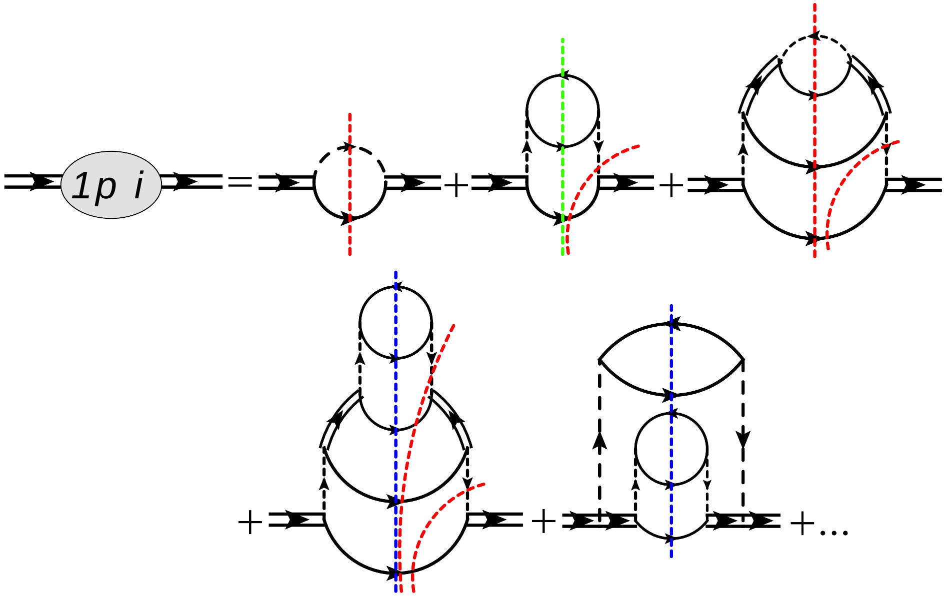




- Hartree-Fock+ improved $2p2h$+ polarisation...

Nieves improvements:

- For all $\Delta - h$ excitations selfenergy model from NPA 468!

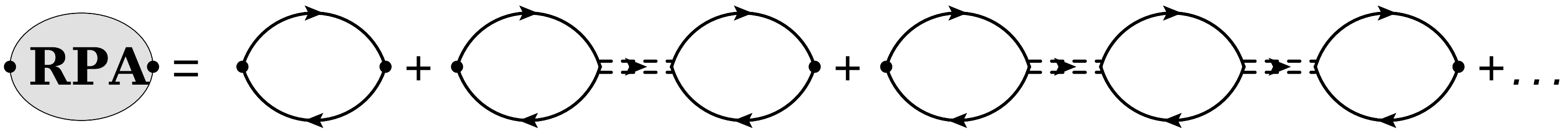


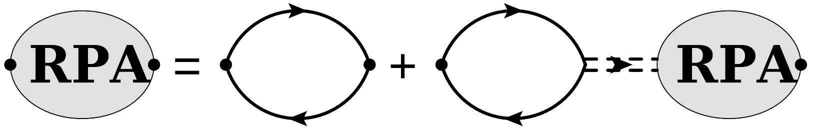
- Physical channels:

$\Im\Sigma_\Delta = \Im\Sigma_{QEL} + \frac{1}{2}\tilde{\Gamma} + \Im\Sigma_{A2} + \Im\Sigma_{A3}$. $\frac{1}{2}\tilde{\Gamma}$ - $\Delta \to \pi N$.

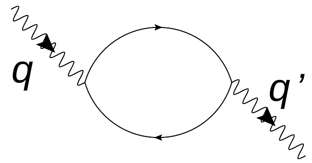
RPA

- Basic idea behind the RPA summation:





- Two possible approaches to polarisation tensor:



1. Because of finite size effects $\Pi^0 = \Pi^0(\omega, \boldsymbol{q}, \boldsymbol{q'})$, $\boldsymbol{q} \neq \boldsymbol{q'}$ (Martini-Marteu). Problem with Feynman analysis, momentum not conserved.
2. Nucleus as an uncorrelated sum of Fermi Seas (LDA) $\Pi^0 = \Pi^0(\omega, \boldsymbol{q})$, $\boldsymbol{q} = \boldsymbol{q'}$ (Nieves-Oset).

RPA by Martini-Marteau

- All nuclear response functions:

$$R(\omega, \boldsymbol{q}) = -\frac{\Omega}{\pi} \Im \Pi(\omega, \boldsymbol{q}, \boldsymbol{q})$$

- One-loop polarisation propagator (ph or $\Delta - h$) in the angular momentum basis:

$$\begin{aligned} \Pi^{(0)J}(\omega, q, q') &= 2\pi \int du P_J(u) \Pi^{(0)}(\omega, \boldsymbol{q}, \boldsymbol{q'}) \\ u &= \cos(\hat{q}, \hat{q}') \end{aligned}$$

- The example RPA sum (nucleon-nucleon interaction):

$$\Pi^J_{NN}(\omega, q, q') = \Pi^{(0)J}_{Nh}(\omega, q, q') + \int \frac{d^3p}{(2\pi)^3} \Pi^{(0)J}_{Nh}(\omega, q, p) V^{NN}(P) \Pi^J_{NN}(\omega, p, q')$$

- Disadvantage: angular momentum basis, matrix integral equations

RPA by Martini-Marteau

- One of the potentials (NN):

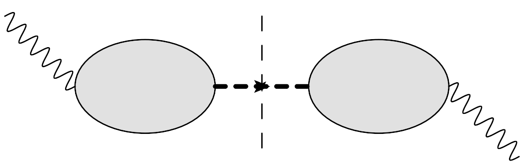
$$\begin{aligned} V_{NN} &= (f' + V_\pi + V_\rho + V_g')\boldsymbol{\tau}^{(1)} \cdot \boldsymbol{\tau}^{(2)} \\ V_\pi &= \left(\frac{g_r}{2M}\right)^2 F_\pi^2(q^2)\frac{\boldsymbol{q}^2}{q^2 - m_\pi^2 + i\epsilon}\boldsymbol{\sigma}^{(1)}\hat{q} \cdot \boldsymbol{\sigma}^{(2)}\hat{q} \\ V_\rho &= \left(\frac{g_r}{2M}\right)^2 C_\rho F_\rho^2(q^2)\frac{\boldsymbol{q}^2}{q^2 - m_\rho^2 + i\epsilon}\boldsymbol{\sigma}^{(1)} \times \hat{q} \cdot \boldsymbol{\sigma}^{(2)} \times \hat{q} \\ V_{g'} &= \left(\frac{g_r}{2M}\right)^2 F_\pi^2 g' \boldsymbol{\sigma}^{(1)} \cdot \boldsymbol{\sigma}^{(2)} \end{aligned}$$

- Two sources of the imaginary part:

$$\boldsymbol{\Im\Pi = |\pi^2|\Im V + |1 + \Pi V|^2 \Im\Pi^{(0)}}$$

- Imaginary part of the potential: π or ρ go on-shell
- Advantage of resigning on momentum conservation:

Coherent pion production!



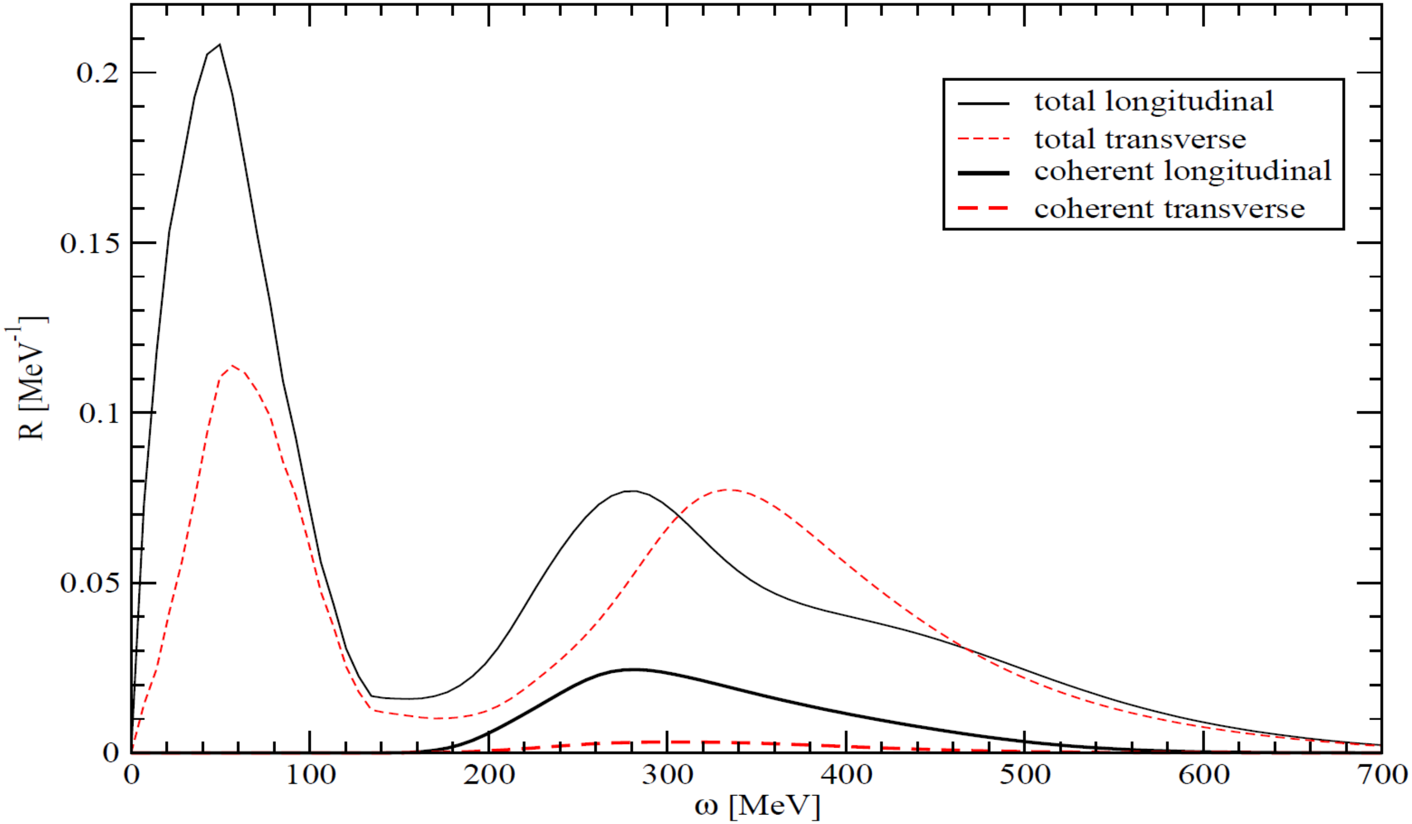
RPA by Nieves:

- At vertices momentum exactly conserved $\rightarrow$ no coherent process, but also no integral equations. RPA sum purely algebraic! (coherent π: other papers by Nieves, but not default inclusion in this model).
- Also Nh and Δh bubbles, forward and backward.
- Due to huge $N\pi\Delta$ and $N\rho\Delta$ interaction uncertainties no RPA for the Delta peak.

Theoretical part summary:

	Martini	Nieves
Basic Feynman diagrams	4: Nucleon pole, Nucleon crossed, Δ pole, Δ crossed	All 7 for π+4 with ρ
Basic polarisation tensors	7?	At least 150!
Relativistic?	No	No
Δ selfenergy	NPA 468	NPA 468
RPA loops	Δ-h, N-h	Δ-h, N-h
RPA range	QEL, DIP, Δ	QEL, DIP
RPA type	Integral equations, angular momentum basis	Algebraic equations, momentum basis
Coherent π?	Yes	No
Medium SF?	Yes, 2p2h	Yes, H-F, 2p2h, polarisation, higher order 2p2h series

Coherent π by Martini



- Longitudinal and transverse responses for ^{12}C and $q = 300[MeV]$. High longitudinal coherent response in the Δ region (PRC 81, 2010).

Different channels by Nieves

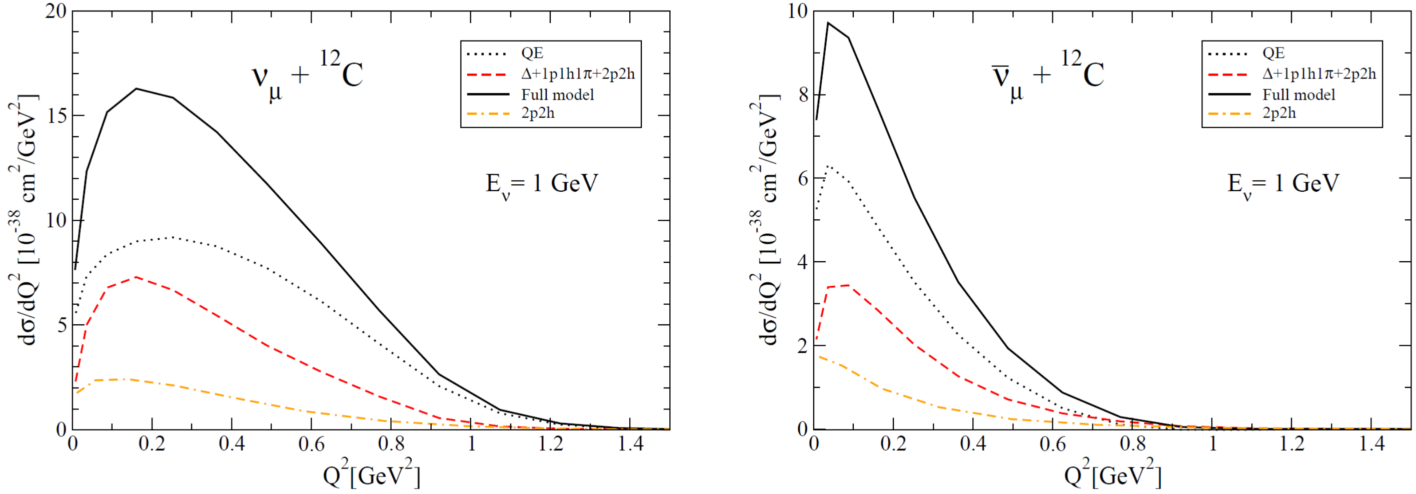
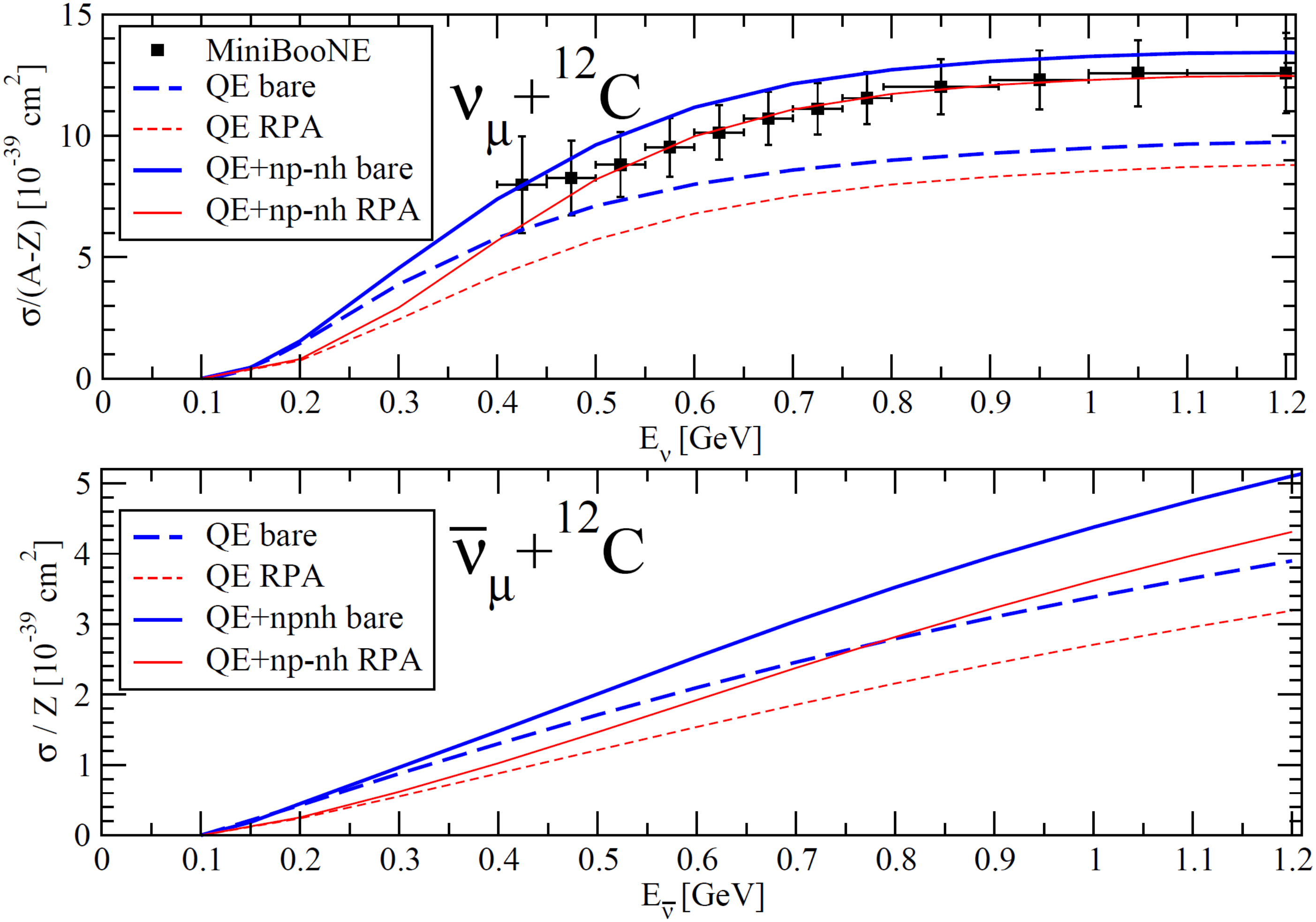


FIG. 17: Muon neutrino (left) and antineutrino (right) CC differential cross section $\frac{d\sigma}{dQ^2}$ in carbon for an incident neutrino energy of 1 GeV ($Q^2 = -q^2$). Different contributions are displayed, standing the solid lines for our full model results.

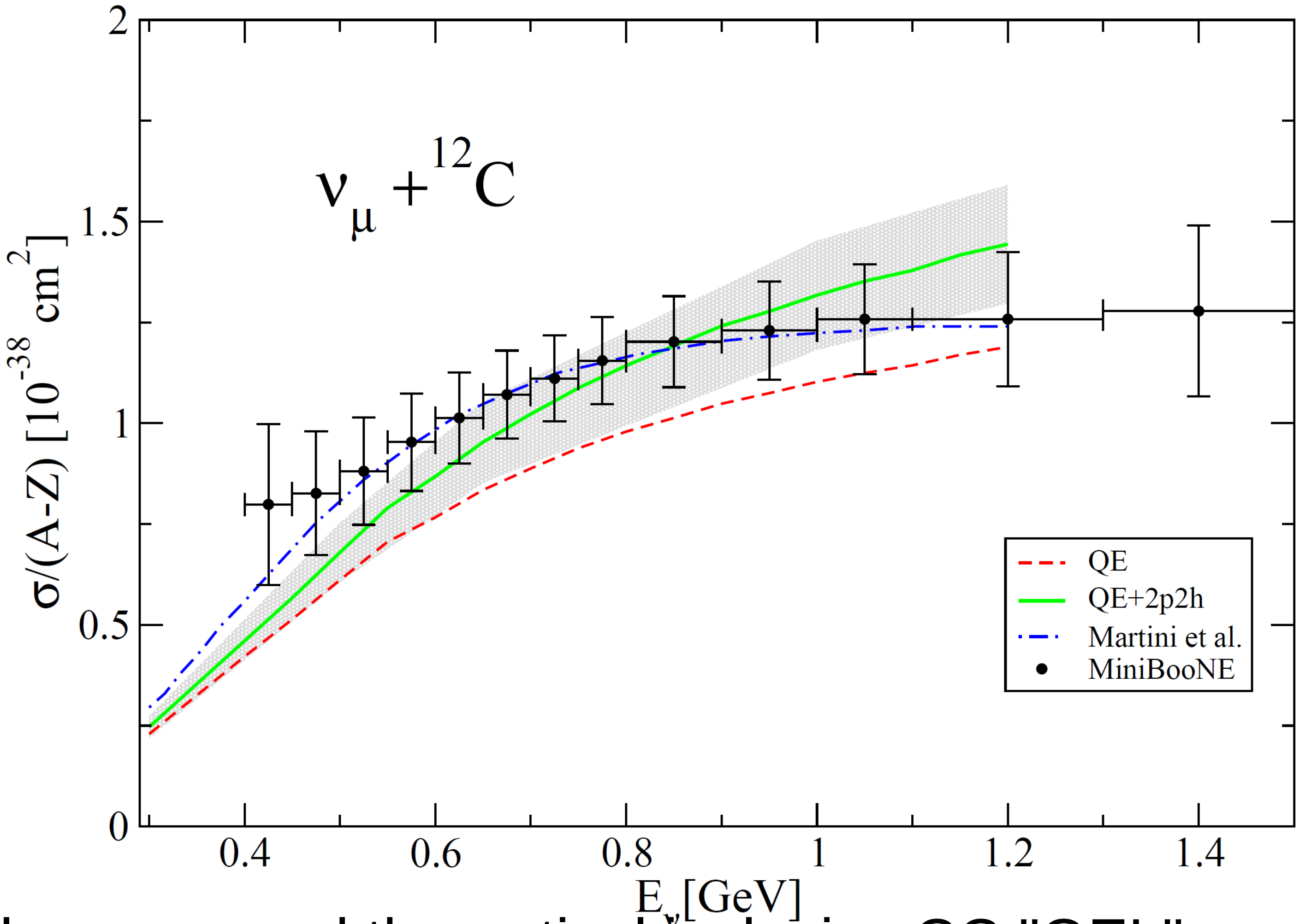
- None of the listed contributions negligible (arXiv:1102.2777v1 [hep-ph]).

Martini vs. MiniBooNE



- Flux-averaged theoretical inclusive CC "QEL" cross-section vs. MiniBooNE data. $np-nh$ contribution crucial, RPA resumations important. No need for high M_A! (PRC 81, 2010)

Nieves vs. MiniBooNe



- Flux-averaged theoretical inclusive CC "QEL" cross-section vs. MiniBooNe data. Again $2p-2h$ contribution crucial and no need for high M_A! (arXiv:1102.2777v1 [hep-ph])

Summary

- Promising results in both models.
- Inclusion of as many as possible dynamical effects crucial.
- Nieves model more sophisticated in number of diagrams (MEC!) but coherent π by default in Martini-Marteau.
- MiniBooNe M_A puzzle solved by the $np - nh$ effects?

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Thank you!

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