Do recent results on neutrino oscillations falsify the Standard Model?!

Jan T. Sobczyk

Institute for Theoretical Physics
Wrocław University

October 11, 2010

Plan of the seminar

Current Section

Motivation for this seminar:

- The investigation of neutrino oscillations started with the Davis solar neutrino Homestake experiment.
- After SuperKamiokande reported atmospheric neutrino oscillations signal several new experiments have been launched.
- For many years the situation was *boring*: all the results could be accomodated in the Standard Model.
- There are two new oscillation experimental results which if confirmed demostrate that the Standard Model in incomplete.

Current Section

We need a theoretical scheme in order to understand the data.

We assume that there are three families with Dirac (for oscillation analysis they can be Majorana as well) neutrinos. The states with well defined flavour are linear combinations of states with well defined mass:

$$|\nu_l>=\sum_m U_{lm}|\nu_m> .$$

In the textbook derivation of the vacuum oscillation formula, we assume that neutrino has well defined momentum $\vec{p}=(p,0,0)$ and thus various mass states have different energies (and velocities!).

Because $E_m \approx p + \frac{M_m^2}{2p}$:

$$|\nu_l(x,t)> = \sum_m U_{lm}|\nu_m(0)> \mathrm{e}^{-i(E_m t - px)}$$

$$\approx \mathrm{e}^{ip(x-t)} \sum_m U_{lm}|\nu_m(0)> \mathrm{e}^{-i\frac{M_m^2 t}{2p}},$$

$$P(\nu_l \to \nu_k, x) = | < \nu_k(x,t)|\nu_l(0,0) > |^2$$

$$P(\nu_l \to \nu_k; L) = \sum_m |U_{km}|^2 |U_{lm}|^2$$

$$+2 \sum_{m>m'} |U_{km} U^*_{lm} U^*_{km'} U_{lm'}| \cos\left(\frac{L(M_m^2 - M_{m'}^2)}{2p} - \Phi_{k,l;m,m'}\right),$$

where $\Phi_{k,l;m,m'} = \arg\left(U_{km} U^*_{lm} U^*_{km'} U_{lm'}\right)$.

$$P(\bar\nu_l \to \bar\nu_k; L) = \sum_m |U_{km}|^2 |U_{lm}|^2$$

$$+2 \sum_{m>m'} |U_{km} U^*_{lm} U^*_{km'} U_{lm'}| \cos\left(\frac{L(M_m^2 - M_{m'}^2)}{2p} + \Phi_{k,l;m,m'}\right),$$

Oscillations occur only if neutrinos are massive.

We restrict to two families only. A convenient way to discuss the oscillations::

$$i\frac{d}{dt}\left(\begin{array}{c}\nu_1\\ \nu_2\end{array}\right)=\hat{H}\left(\begin{array}{c}\nu_1\\ \nu_2\end{array}\right),$$

$$\hat{H}=\left(\begin{array}{cc}E_1 & 0\\ 0 & E_2\end{array}\right)\approx\left(\begin{array}{cc}p+\frac{M_1^2}{2E} & 0\\ 0 & p+\frac{M_2^2}{2E}\end{array}\right)$$

$$=\left(p+\frac{M_1^2+M_2^2}{4E}\right)\left(\begin{array}{cc}1 & 0\\ 0 & 1\end{array}\right)-\frac{\Delta M_{12}^2}{4E}\left(\begin{array}{cc}1 & 0\\ 0 & -1\end{array}\right).$$

Only the non-diagonal part is relevent for oscillations.

The Hamiltonian in the flavour basis $\hat{H}_f$

$$\left(\begin{array}{c}\nu_\nu\\ \nu_\mu\end{array}\right) = U\left(\begin{array}{c}\nu_1\\ \nu_2\end{array}\right), \quad U = \left(\begin{array}{cc}\cos\Theta & \sin\Theta\\ -\sin\Theta & \cos\Theta\end{array}\right),$$

$$\hat{H}_f = U\hat{H}U^{-1} = \left(p + \frac{M_1^2 + M_2^2}{4E}\right)\left(\begin{array}{cc}1 & 0\\ 0 & 1\end{array}\right)$$

$$-\frac{\Delta M^2}{4E}\left(\begin{array}{cc}\cos 2\Theta & -\sin 2\Theta\\ -\sin 2\Theta & -\cos 2\Theta\end{array}\right)$$

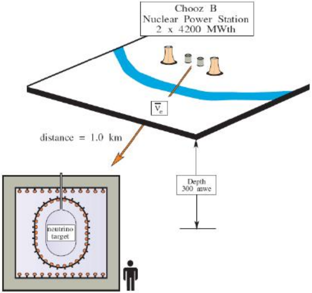
The non-diagonal part gives rise to the oscillation pattern:

$$P(\nu_e \to \nu_\mu; L) = \sin^2 2\Theta \sin^2 \frac{L\Delta M^2}{4E}.$$

The wave packets analysis leads to the same results and we accept the standard theory.

But we must add matter effects which are very importants.

Before we do that, we present the typical oscillation analysis.



For a long time the best estimation of Θ_{13} was coming from the reactor neutrinos CHOOZ experiment.

No oscillations were seen with the accuracy of 5%:

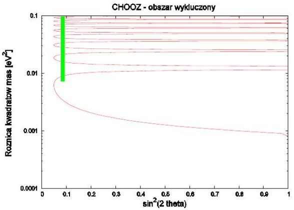
$$\sin^2 2\Theta \sin^2 1.27 \frac{\Delta M^2 [GeV^2] L[km]}{E[GeV]} < 0.05.$$

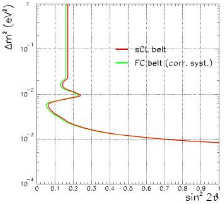
The experiment is characterized by:
$L \cong 1$ km, $E \cong 5 \cdot 10^{-3}$ GeV.

On the left below we see the contour:

$$\sin^2 2\Theta \sin^2 1.27\frac{\Delta M^2[GeV^2]L[km]}{E[GeV]} = 0.05.$$

The excluded region is on the right from the curve. Because neutrinos are not monoenergetic only few oscillation minima and maxima can be seen.
On the right below we show the excluded region from the actual analysis.





Each experiment is sensitive to some values of ΔM^2. From the basic oscillation formula

$$\min(\Delta M^2) \sim \frac{2 < E >}{L}.$$

Table 13.1: Sensitivity of different oscillation experiments.

Source	Type of ν	$\overline{E}$[MeV]	L[km]	$\min(\Delta m^2)$[eV^2]
Reactor	$\overline{\nu}_e$	~ 1	1	$\sim 10^{-3}$
Reactor	$\overline{\nu}_e$	~ 1	100	$\sim 10^{-5}$
Accelerator	$\nu_\mu, \overline{\nu}_\mu$	$\sim 10^3$	1	~ 1
Accelerator	$\nu_\mu, \overline{\nu}_\mu$	$\sim 10^3$	1000	$\sim 10^{-3}$
Atmospheric ν's	$\nu_{\mu,e}, \overline{\nu}_{\mu,e}$	$\sim 10^3$	10^4	$\sim 10^{-4}$
Sun	ν_e	~ 1	1.5×10^8	$\sim 10^{11}$

Typically, for each experiment one works out an approximation with a dominant 2D oscillation pattern.
There has been also a lot of research on the full 3D oscillation parameters pattern.

In matter neutrinos are subject to scattering and absorption. The main effect is elastic forward scattering with coherently summed scattered waves. As a result, a refraction index does appear:

$$n_\alpha - 1 = \sum_j \frac{f_\alpha^j(0) \cdot N_j}{k^2},$$

$\alpha = e, \mu, \tau$, $f_\alpha^j(\vartheta)$ is the amplitude of ν_α scattering in the angle ϑ on j component of the matter with density N_j.

The refraction indices change the phase velocities of neutrino waves. If matter is nonsymmetric with respect to neutrino falavour states, the additional phase difference appears:

$$\Delta\phi = k(n_e - n_\mu)\cdot t = \sum_j \Delta \frac{f^j(0)\cdot N_j}{k} t.$$

The effect comes from different ν_e and ν_μ interactions with electrons.

$$\Delta\phi = \sqrt{2} G_F N_e t.$$

It is useful to introduce the effective potential (or strictly speaking the difference of potentials):

$$V = \sqrt{2} G_F N_e.$$

In the matter the Hamiltonian becomes:

$$\hat{H}_f^{matt} = \left(p + \frac{M_1^2 + M_2^2}{4E}\right)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$-\frac{\Delta M^2}{4E}\begin{pmatrix} \cos 2\Theta & -\sin 2\Theta \\ -\sin 2\Theta & -\cos 2\Theta \end{pmatrix} + \frac{V}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{V}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The non-diagonal part, which is responsible for oscillations, can be written as ($\eta = \frac{V/2}{\Delta M^2/4E}$)

$$\frac{\Delta M^2}{4E}\begin{pmatrix} -(\cos 2\Theta - \eta) & \sin 2\Theta \\ \sin 2\Theta & \cos 2\Theta - \eta \end{pmatrix} =$$

$$= \frac{\Delta M^2_{matt}}{4E}\begin{pmatrix} -\cos 2\Theta_{matt} & \sin 2\Theta_{matt} \\ \sin 2\Theta_{matt} & \cos 2\Theta_{matt} \end{pmatrix},$$

where

$$\sin 2\Theta_{matt} = \frac{\sin 2\Theta}{\sqrt{\sin^2 2\Theta + (\cos 2\Theta - \eta)^2}},$$

$$\Delta M^2_{matt} = \Delta M^2 \sqrt{\sin^2 2\Theta + (\cos 2\Theta - \eta)^2}.$$

It shouls be clear that in the matter we get the identical oscillation formula, however with different parameters

$$\Theta \to \Theta_{matt}, \qquad \Delta M^2 \to \Delta M^2_{matt}.$$

From the CPT invariance

$$P(\nu_x \rightarrow \nu_y; L) = P(\bar{\nu}_y \rightarrow \bar{\nu}_x; L)$$

and in particular

$$P(\nu_x \rightarrow \nu_x; L) = P(\bar{\nu}_x \rightarrow \bar{\nu}_x; L)$$

The study of disappearance in vacuum tells us nothing about CP violation.

The proper measure of CP asymmetry:

$$A_{CP}^{(l'l)} = P(\nu_l \rightarrow \nu_{l'}; L) - P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}; L)$$

$$A_{CP}^{(l'l)} = 4 \sum_{m>m'} (U_{l'm} U_{lm}^* U_{lm'}^* U_{l'm'}) \sin \frac{M_m^2 - M_{m'}^2}{2p} L.$$

$$A_{CP}^{(\mu e)} = -A_{CP}^{(\tau e)} = A_{CP}^{(\tau\mu)}$$

$$= 4J_{CP}\left(\sin\frac{M_3^2 - M_2^2}{2p}L + \sin\frac{M_2^2 - M_1^2}{2p}L + \sin\frac{M_1^2 - M_3^2}{2p}L\right).$$

$$J_{CP} = \Im\left(U_{\mu 3}U_{e3}^{*}U_{e2}U_{\mu 2}^{*}\right)$$

If any two masses are equal there is no CP violation!

Matter effects have important impact on CP violation-like effects.

- The matter is not C invariant (it contains e^- and not e^+).
- For antineutrinos the effective potential changes sign: $N_e \longrightarrow -N_e$.
- With the matter effects there can be different ΔM^2_{matt} for neutrinos and for antineutrinos.

Current Section

We assume three families of Dirac/Majorana massive neutrinos. The general form of the Pontecorvo-Maki-Nakagawa-Sakata mixing matrix is:

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \times \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix}$$

$$\times \begin{bmatrix} c_{12} & {}_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Th last factor (phases $\alpha_{1,2}$) is present for Majorana neutrinos only.

The oscillation formula contains three mixing angles: Θ_{12}, Θ_{13}, Θ_{23} and two independent differences of squares of masses $\Delta^2_{jk} = M^2_j - M^2_k$. $\alpha_{1,2}$ do not enter the formula.
The conventional ordering of flavours is $(\nu_e, \nu_\mu, \nu_\tau)$.

Atmospheric neutrinos (later on confirmed in K2K, MINOS long baseline experiments):

$$|\Delta^2_{31}| \cong 2.4 \cdot 10^{-3}\mathrm{eV}^2, \quad \Theta_{23} \cong 39 - 51^o,$$

Solar neutrinos (later on confirmed in KAMLAND reactor neutrino experiment)

$$\Delta^2_{21} \cong 7.6 \cdot 10^{-5}\mathrm{eV}^2, \quad \Theta_{12} \cong 34^o,$$

Finally from CHOZZ reactor neutrino experiment

$$\Theta_{13} < 11^o.$$

There is a very interesting global data analysis which includes the most recent results from the KAMLAND [arXiv 1009.4771 (hep-exp)].

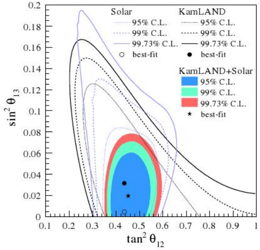


FIG. 3: Allowed regions from the solar and KamLAND data projected in the $(\tan^2\theta_{12}, \sin^2\theta_{13})$ plane for the three-flavor analysis.

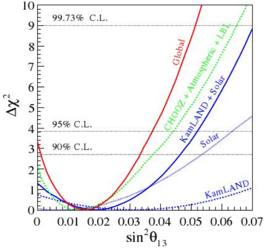


FIG. 4: $\Delta\chi^2$-profiles projected onto the $\sin^2\theta_{13}$ axis for different combinations of the oscillation data floating the undisplayed parameters $(\tan^2\theta_{12}, \Delta m^2_{21})$.

Perhaps we already know the value of Θ_{13}?!
Note that $\sin^2\Theta_{13} \sim 0.017$ translates to $\sin^2 2\Theta_{13} \sim 0.068$.

TABLE III: Summary of the best-fit values for $\tan^2\theta_{12}$ and $\sin^2\theta_{13}$ from two- and three-flavor neutrino oscillation analyses of various combinations of experimental data. "Global" refers to the combined data from the KamLAND, solar, CHOOZ, atmospheric, and long-baseline accelerator experiments.

Data set	Analysis method	$\tan^2\theta_{12}$	$\sin^2\theta_{13}$
KamLAND	two-flavor	$0.492^{+0.086}_{-0.067}$	$\equiv 0$
KamLAND + solar	two-flavor	$0.444^{+0.036}_{-0.030}$	$\equiv 0$
KamLAND	three-flavor	$0.436^{+0.102}_{-0.081}$	$0.032^{+0.037}_{-0.037}$
KamLAND + solar	three-flavor	$0.452^{+0.035}_{-0.033}$	$0.020^{+0.016}_{-0.016}$
Global	three-flavor	$0.452^{+0.033}_{-0.032}$	$0.017^{+0.010}_{-0.009}$

Current Section

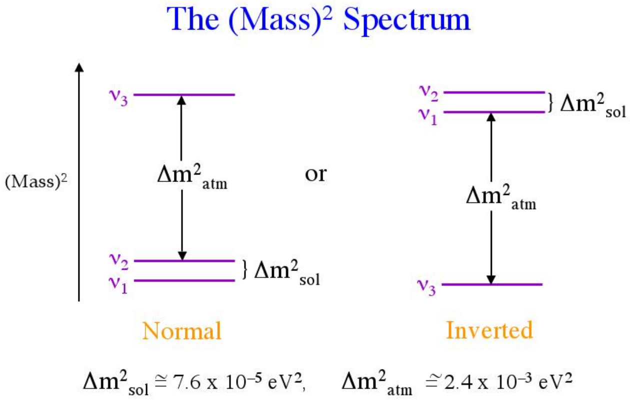
Open questions:

- Θ_{13}?
- absolute mass scale?
- mass hierarchy?
- Dirac/Majorana?
- how many families?

There is a variety of approaches and it is difficult to predict which one will be most successfull.

- direct measurement in tritium β decay
  $< m_\beta >= \sqrt{\sum_j |U_{ej}|^2 m_j^2} < 2eV$
  KATRIN will be sensitive to $m_j \sim 0.35$ eV.
- cosmology, from WMAP and large scale structure
  $\sum_j m_j < (0.4 - 1)$ eV
- $0\nu 2\beta$ decay
  $< m >$ depends on the Majorana phases $\alpha_{1,2}$
- supernova
  SN1987A allowed for the bound of 12 eV.

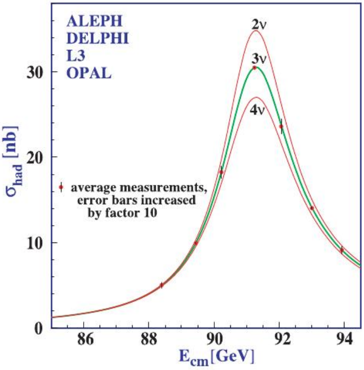
There are two options for the mass hierarchy:



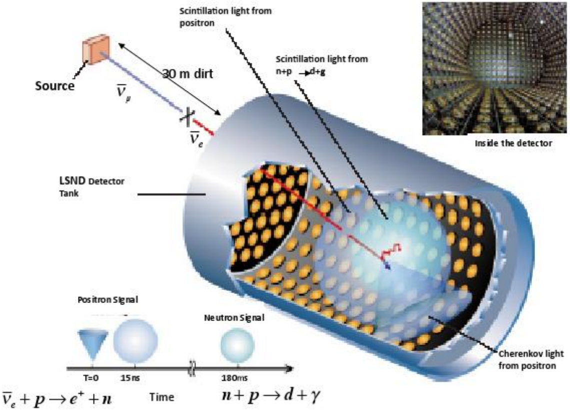
[from B. Kayser]

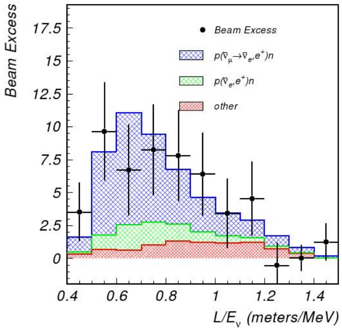
Current Section

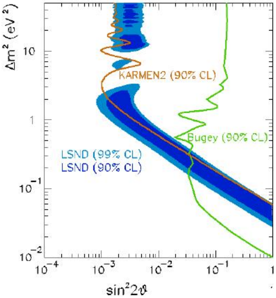
In the Standard Model there are three light neutrino flavour states.



In 1995 LSND reported a puzzling oscillation signal







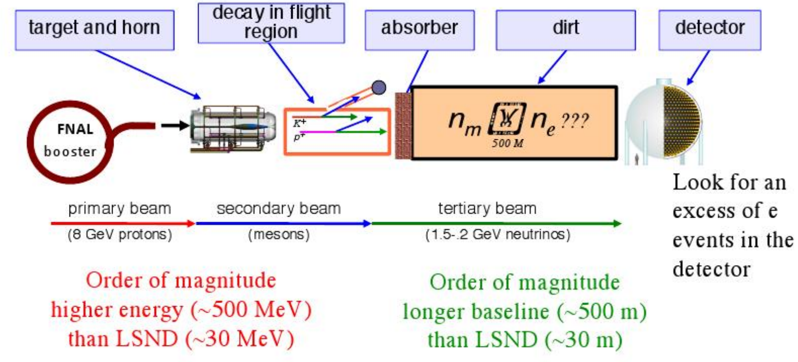
A part of the allowed region was excluded by Burgey and KARMEN experiments.

$\Delta M^2 > 0.2$ eV^2.

3.8σ effect.

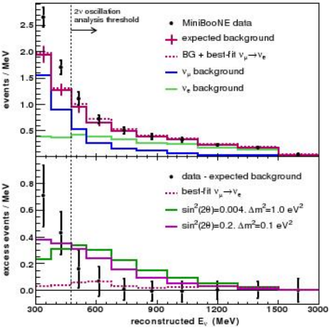
If confirmed > 3 neutrino mass states are necessary!

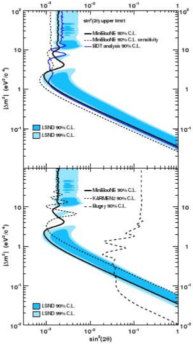
MiniBooNE experiment has been set up at FermiLab in order to investigate the same L/E region.



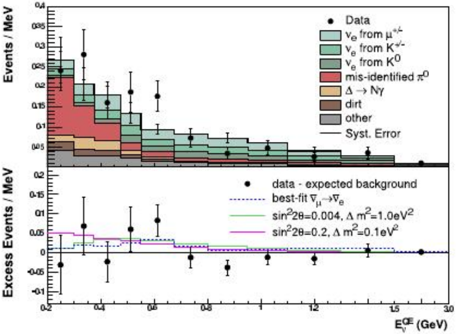
[from G. Garvey]

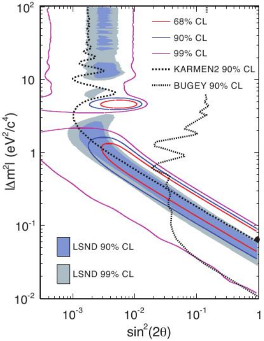
With the neutrino flux no oscillation signal was observed (the integrated ν_μ flux from $6.46E20$ POT).



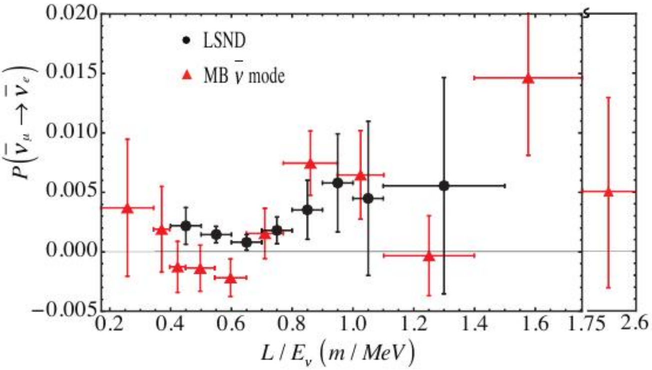


The very recent antineutrino flux data (from 5.67E20 POT) seem to confirm the LSND signal!





It is interesting to put together LSND and MiniBooNE antineutrino data points:



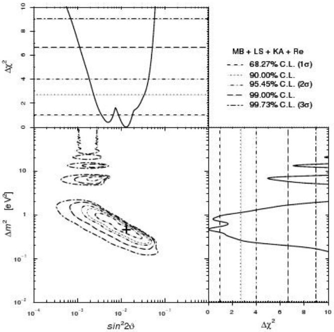


FIG. 7. Allowed regions in the $\sin^2 2\vartheta$–Δm^2 plane and marginal $\Delta\chi^2$'s for $\sin^2 2\vartheta$ and Δm^2 obtained from the combined fit of MiniBooNE (MB), LSND (LS) and KARMEN (KA) $\bar{\nu}_\mu \to \bar{\nu}_e$ data and the exclusion curves obtained from the fit of reactor Bugey and Chooz (Re) $\bar{\nu}_e \to \bar{\nu}_e$ data. The best-fit point is indicated by a cross.

Global analysis of electron anti-neutrino data: Giunti & Laveder, arXiv:1010.1395 [hep-ph].

The data from the LSND, MinBooNE, KARMEN and reactor experiments are in excellent agreement.

Georgia Karagiorgi theory:

3 active + 1 sterile scheme cannot account for an apparent CP violation, in the 2-families approximation (the leading effect):

$$P(\nu_\mu \to \nu_e, L) = 4 \mid U_{e4} \mid^2 \mid U_{\mu 4} \mid^2 \sin^2(1.27 \Delta m^2_{41} L/E),$$

$$P(\nu_\mu \to \nu_e, L) = P(\bar{\nu}_\mu \to \bar{\nu}_e, L).$$

With an extra sterile neutrino i.e. in the 3+2 scheme we get

$$P(\nu_\mu \to \nu_e, L) = 4 \mid U_{e4} \mid^2 \mid U_{\mu 4} \mid^2 \sin^2(1.27 \Delta m^2_{41} L/E)$$

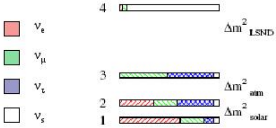
$$+4 \mid U_{e5} \mid^2 \mid U_{\mu 5} \mid^2 \sin^2(1.27 \Delta m^2_{51} L/E)$$

$$+4 \mid U_{e4} \mid \mid U_{\mu 4} \mid \mid U_{e5} \mid \mid U_{\mu 5} \mid \sin(1.27 \Delta m^2_{41} L/E) \sin(1.27 \Delta m^2_{51} L/E)$$

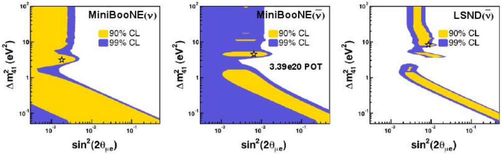
$$\times \cos(1.27 \Delta_{54} L/E - \phi_{54}.$$

For antineutrinos $\phi_{54} \to -\phi_{54}$.

3 active + 1 sterile neutrinos (3+1)



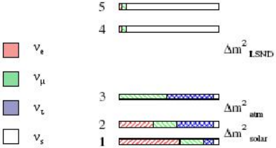
Each of the three datasets fit separately to a (3+1) model yields the following allowed regions:



All three results have low compatibility, at 1.8%,
but two of them (antineutrino) are compatible at 49%.

[from G. Koragiorgi]

3 active + 2 sterile neutrinos (3+2)



Akhmedov & Schwetz theory, arXiv:1007.4171 [hep-ph].

In addition to the standard CC interaction there is a term:

$$\mathcal{L}_{NSI} = -2\sqrt{2}G_F \sum_{\alpha\beta} \epsilon_{\alpha\beta}^{f,f'}(L,R)\left(\bar{f}P_{L,R}\gamma^\mu f'\right)\left(\bar{l}_\alpha P_L \gamma_\mu \nu_\beta\right) + h.c.$$

In the presence of FSI a neutrino produced/detected along with a charged lepton l_α in a process $(f, f') \equiv X$ is a linear combination of flavour eigenstates:

$$\mid \nu_\alpha^X >= C_\alpha^X \left(\mid \nu_\alpha > + \sum_\beta \epsilon_{\alpha\beta}^X \mid \nu_\beta > \right),$$

where C_α^X is the normalization constant.

Akhmedov & Schwetz theory, arXiv:1007.4171 [hep-ph] (cont).

In addition to the standard flavour states there exist also a fourth light sterile neutrino ν_s. As usual

$$\mid \nu_\alpha >= \sum_j U_{\alpha j} \mid \nu_j > .$$

The 3+1 scheme is assumed i.e. $\Delta m^2_{41} \sim 1$ eV^2.

It is possible to achieve

$$P(\nu_\alpha \to \nu_\beta, L) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\beta, L).$$

MiniBooNE plans to double (almost) the statistics to $\sim 10E20$ POT.

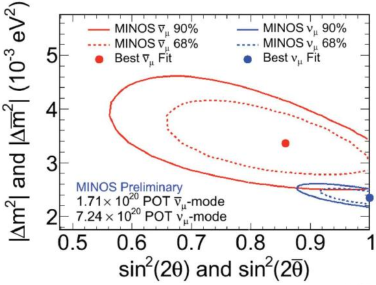
Also new experiments are planned: uBooNE and BooNE.

In Europe there is an idea to put the ICARUS detector near CERN on an off-axis beam.

Current Section

The aim of the MINOS experiment is to investigate the oscillation region of the atmospheric neutrinos.

Recently the antineutrino data were published with unexpected results.



P. Vahle, Neutrino 2010

On the plot there are fits for the oscillation parameters determined by the measurements of $P(\nu_\mu \rightarrow \nu_\mu)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu)$.

The MSW effect with standard interactions cannot explain the results.

Kopp, Machado, Parke theory (arXiv:1009.0014 [hep-ph])

Non-standard NC interactions are considered:

$$\mathcal{L}_{NSI} = -2\sqrt{2}G_F\varepsilon_{\alpha\beta}[\bar{f}\gamma^\mu Pf][\bar{\nu}_\alpha\gamma_\mu P_L\nu_\beta].$$

The authors confine to two family ν_μ and ν_τ system.

The effective Hamiltonian is:

$$H_{eff} = -\frac{\Delta M^2}{4E}\begin{pmatrix} \cos 2\Theta & -\sin 2\Theta \\ -\sin 2\Theta & -\cos 2\Theta \end{pmatrix} + \\ +\frac{A}{2E}\begin{pmatrix} \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau} \\ \varepsilon^*_{\mu\tau} & \varepsilon_{\tau\tau} \end{pmatrix},$$

$A = 2\sqrt{2}G_F N_e E$.

The survival probability is:

$$P(\nu_\mu \to \nu_m u) = 1 - \frac{\mid \Delta M^2 \sin 2\Theta + 2\varepsilon_{\mu\tau} A \mid^2}{\Delta M_N^4} \sin^2\left(\frac{\Delta M_N^2 L}{4E}\right),$$

$$\Delta M_N^2 = \sqrt{(\Delta M^2 \cos 2\Theta + \varepsilon_{\tau\tau} A)^2 + \mid \Delta M^2 \sin 2\Theta + 2\varepsilon_{\mu\tau} A \mid^2}.$$

For antineutrinos $\varepsilon_{\mu\tau} \to \varepsilon_{\mu\tau}^*$ and $A \to -A$.

Thus $P(\nu_\mu \to \nu_\mu) \neq P(\bar{\nu}_\mu \to \bar{\nu}_\mu)$.

Heeck, Redejohann theory (arXiv:1007.2655 [hep-ph])

- Extra $U(1)$ gauge symmetry is introduced
- $L_\mu - L_\tau$ is gauged and the theory is anomaly free

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{Z'} + \mathcal{L}_{mix}$$

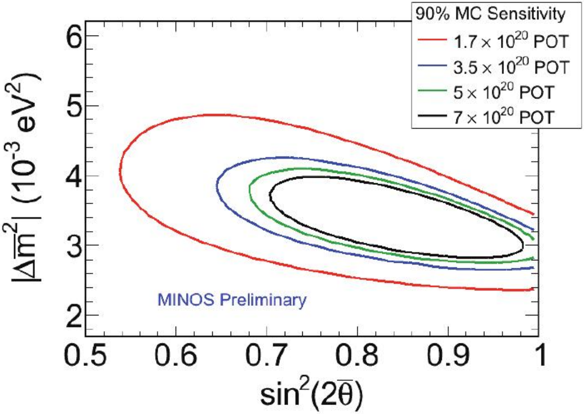
$$\mathcal{L}_{Z'} = -\frac{1}{4}\hat{Z'}_{\mu\nu}\hat{Z'}^{\mu\nu} + \frac{1}{2}\hat{M'}^2_Z\hat{Z'}_\mu\hat{Z'}^\mu - \hat{g}'j'^\mu\hat{Z'}_\mu,$$

$$j'^\mu = \bar{\mu}\gamma^\mu\mu + \bar{\nu}_\mu\gamma^\mu P_L\nu_\mu - \bar{\tau}\gamma^m u\tau - \bar{\nu}_\tau\gamma^\mu P_L\nu_\tau.$$

The term $\frac{1}{2}\hat{M'}^2_Z\hat{Z'}_\mu\hat{Z'}^\mu$ is generated by an unspecified Higgs sector.

$$\mathcal{L}_{mix} = -\frac{\sin\chi}{2}\hat{Z'}^{\mu\nu}\hat{B}_{\mu\nu} + \delta M^2\hat{Z'}_\mu\hat{Z}^\mu.$$

MINOS will have more data and better precison:



[from P. Vahle]

Current Section

Conclusions:

- The recent MiniBooNE and MINOS antineutrino oscillation results can open a window to a physics beyond SM.
- Good time for theorists: models with extra sterile neutrinos and/or extra interactions are testable
- It is important to have better statistics results and also confirmations from other experiments.