S-MATRIX APPROACH TO HADRON GAS

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University of Wroclaw

IFT NEUTRINO SEMINAR
15 MAY 2017

CONTENT

- QCD equation of state
- S-matrix approach to broad resonances
- extension to N-body

QCD EQUATION OF STATE

HADRON RESONANCE GAS MODEL

- Confinement

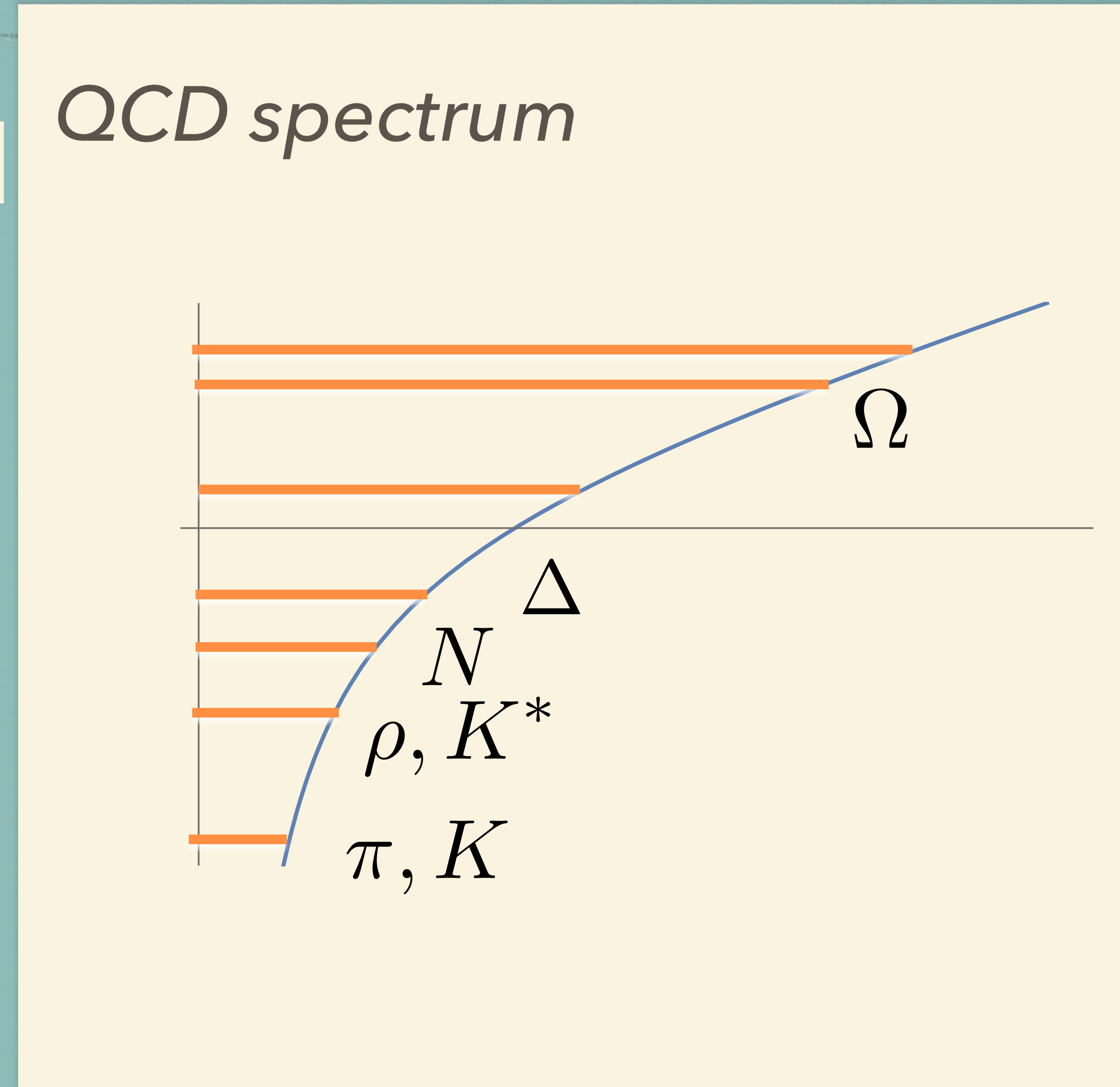
physical quantities $\Rightarrow$ hadronic states representation

$$Z = \sum_{\alpha = B, M} \langle \alpha | e^{-\beta H} | \alpha \rangle$$

HADRON RESON
MODEL

- Confinement

physical quantities



$$Z = \sum_{\alpha = B, M} \langle \alpha | e^{-\beta H} | \alpha \rangle$$

HADRON RESONANCE GAS MODEL

- Ground states $\pi, K, P, N...$

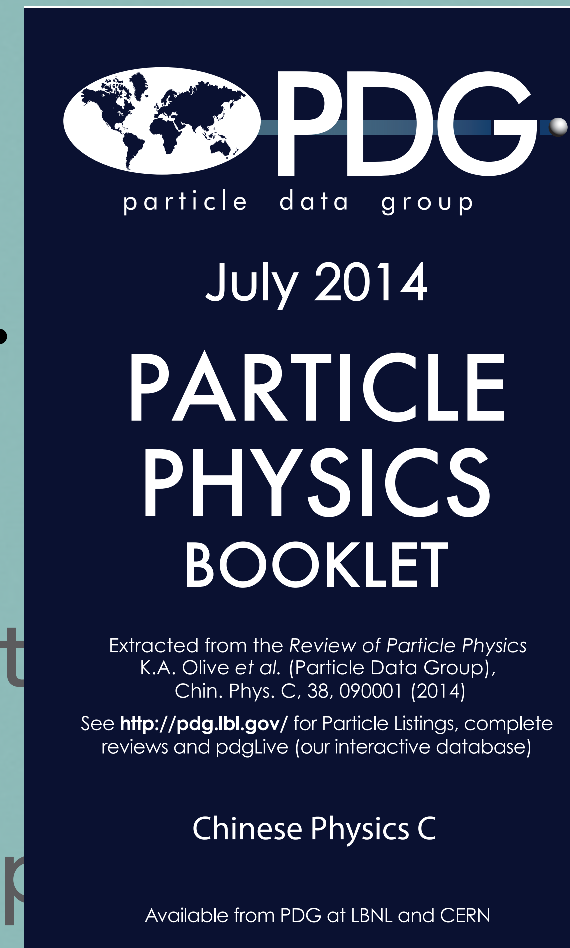
- Resonance formation dominates thermodynamics

- Resonances treated as point-like particles

$$P = T \sum_{\alpha=M,B} g_\alpha \int \frac{d^3k}{(2\pi)^3} \mp \ln(1 \mp e^{-\beta\sqrt{k^2+M_\alpha^2}})$$

HADRON RESONANCE GAS MODEL

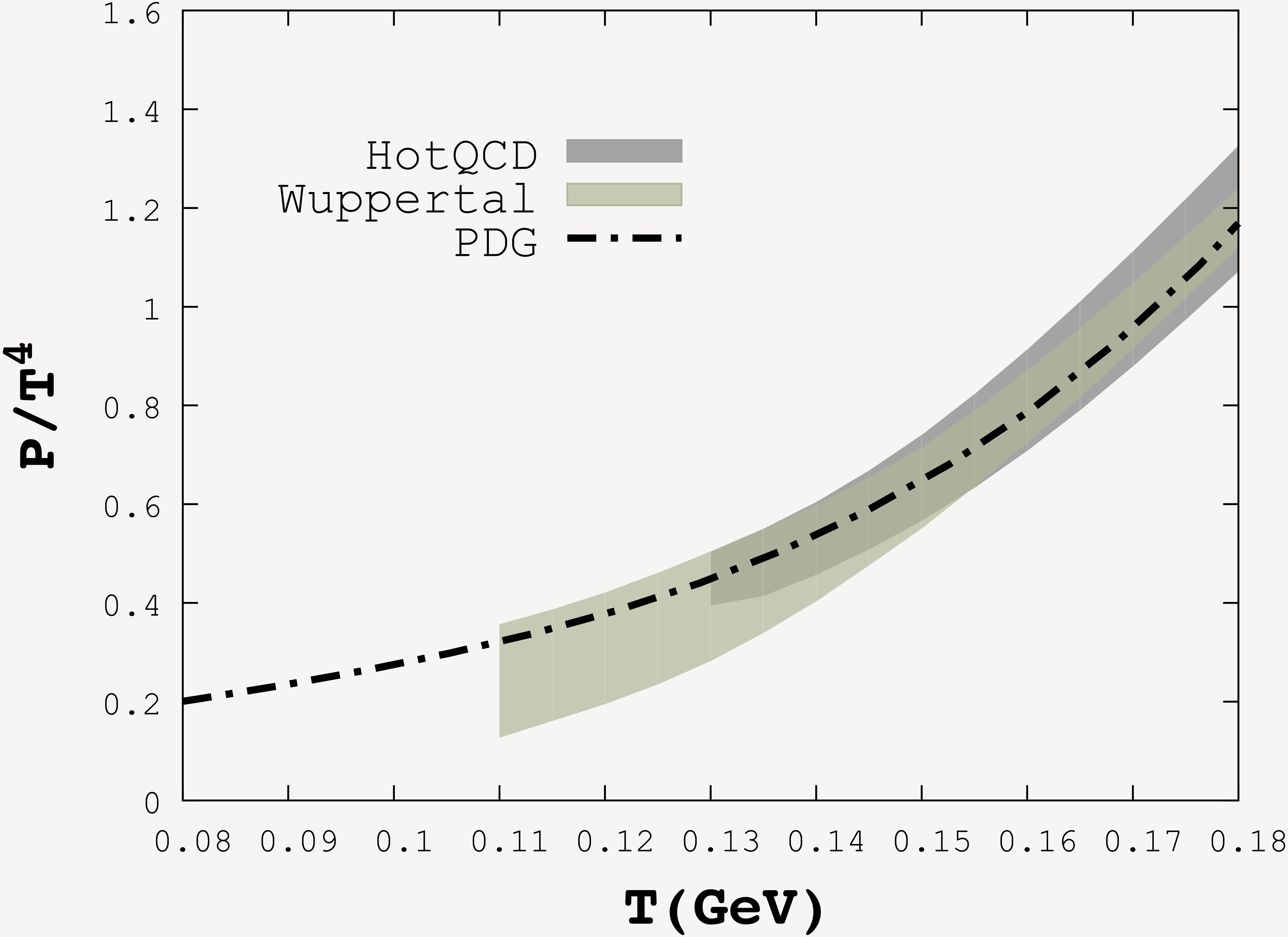
- Ground states $\pi, K, P, N...$


PDG
particle data group
July 2014
PARTICLE PHYSICS BOOKLET
Extracted from the *Review of Particle Physics*
K.A. Olive *et al.* (Particle Data Group),
Chin. Phys. C, 38, 090001 (2014)
See **http://pdg.lbl.gov/** for Particle Listings, complete reviews and pdgLive (our interactive database)
Chinese Physics C
Available from PDG at LBNL and CERN

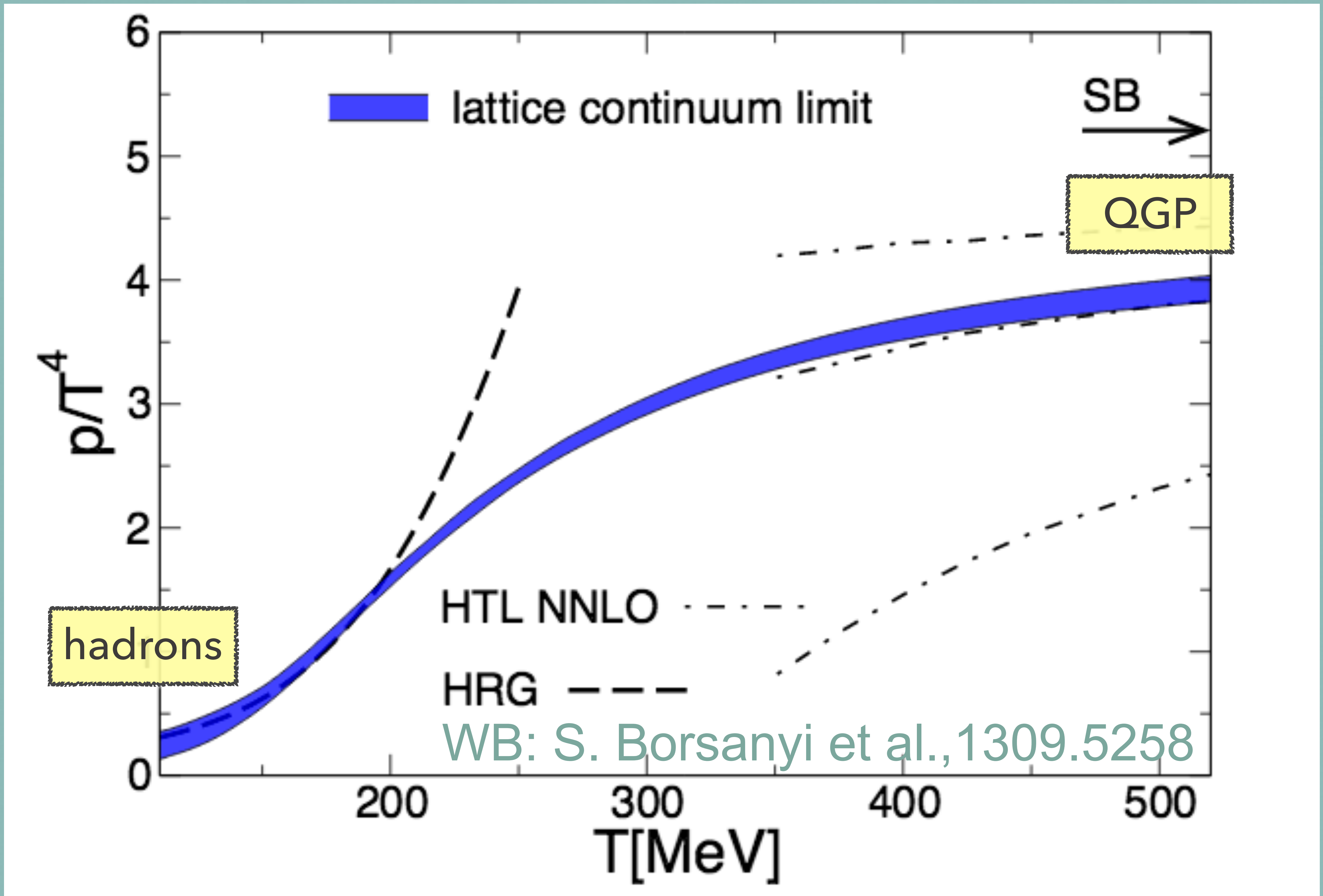
- Resonance formation dominates t nics

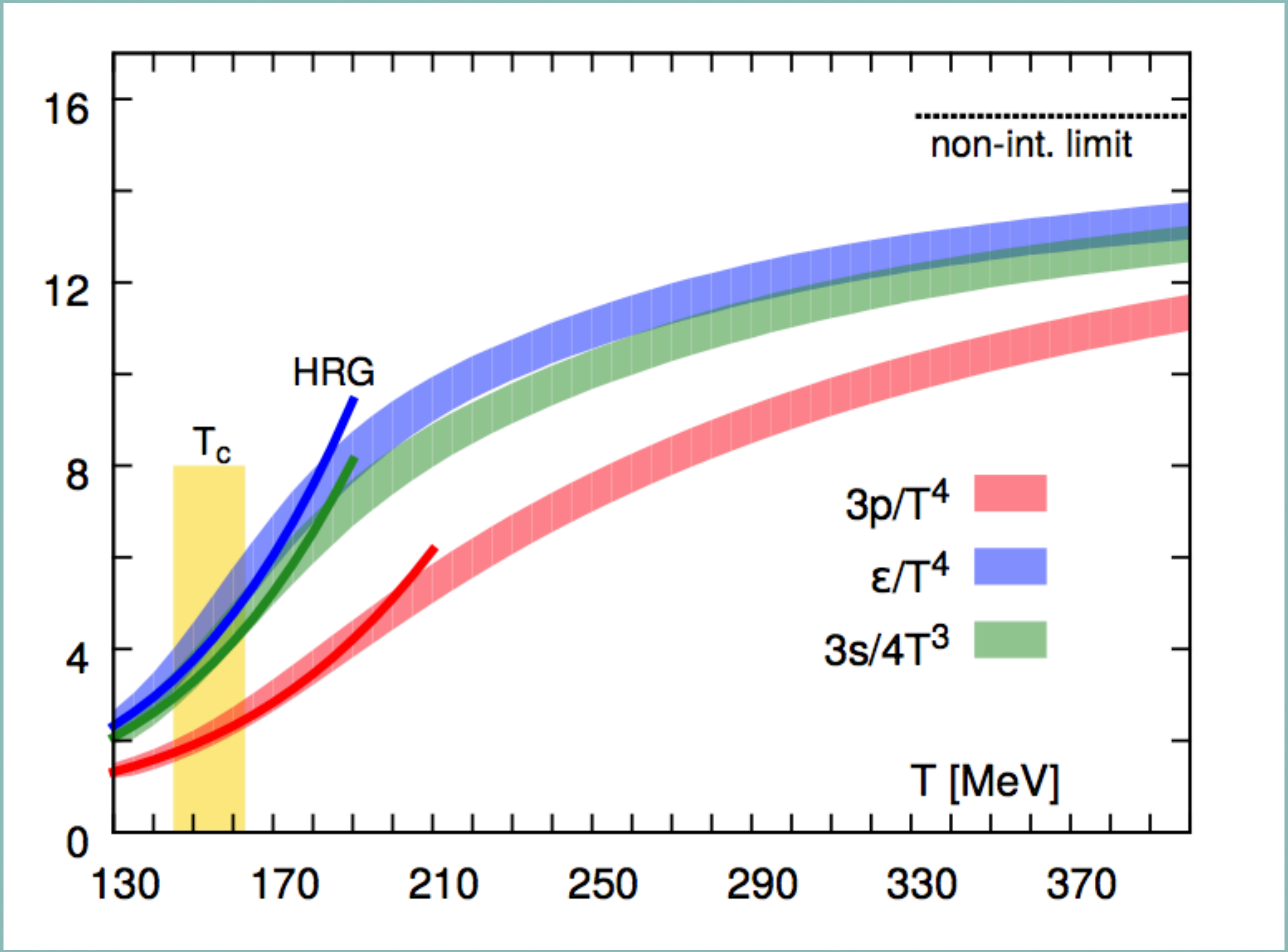
- Resonances treated as point-like p

$$P = T \sum_{\alpha=M,B} g_{\alpha} \int \frac{d^3k}{(2\pi)^3} \mp \ln(1 \mp e^{-\beta\sqrt{k^2+M_{\alpha}^2}})$$



HotQCD
Wuppertal
PDG
P/T^4
T(GeV)
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0.08
0.09
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18


lattice continuum limit
SB
QGP
HTL NNLO
HRG
hadrons
WB: S. Borsanyi et al.,1309.5258
p/T^4
T[MeV]
0
2
3
4
5
6
200
300
400
500



A. Bazavov et al. (hotQCD), PRD90 (2014) 094503

FLUCTUATIONS

- studying the system by linear response

$$\mu = \mu_B B + \mu_Q Q + \mu_S S$$

$$\chi_{B,S,\ldots} = \frac{1}{\beta V} \frac{\partial^2}{\partial \bar{\mu}_B \partial \bar{\mu}_S \ldots} \ln Z$$

μ_B

μ_S

μ_Q

m_q

FLUCTUATIONS

- Baryon sector

$$P = T \sum_{\alpha=\cancel{M},B} g_\alpha \int \frac{d^3k}{(2\pi)^3} \mp \ln(1 \mp e^{-\beta\sqrt{k^2+M_\alpha^2}})$$

or introduce the chemical potential

$$P = T \sum_{\alpha=B,\bar{B}} g_\alpha \int \frac{d^3k}{(2\pi)^3} \ln(1 + e^{-\beta\sqrt{k^2+M_\alpha^2} \pm \bar{\mu}_B})$$

FLUCTUATIONS

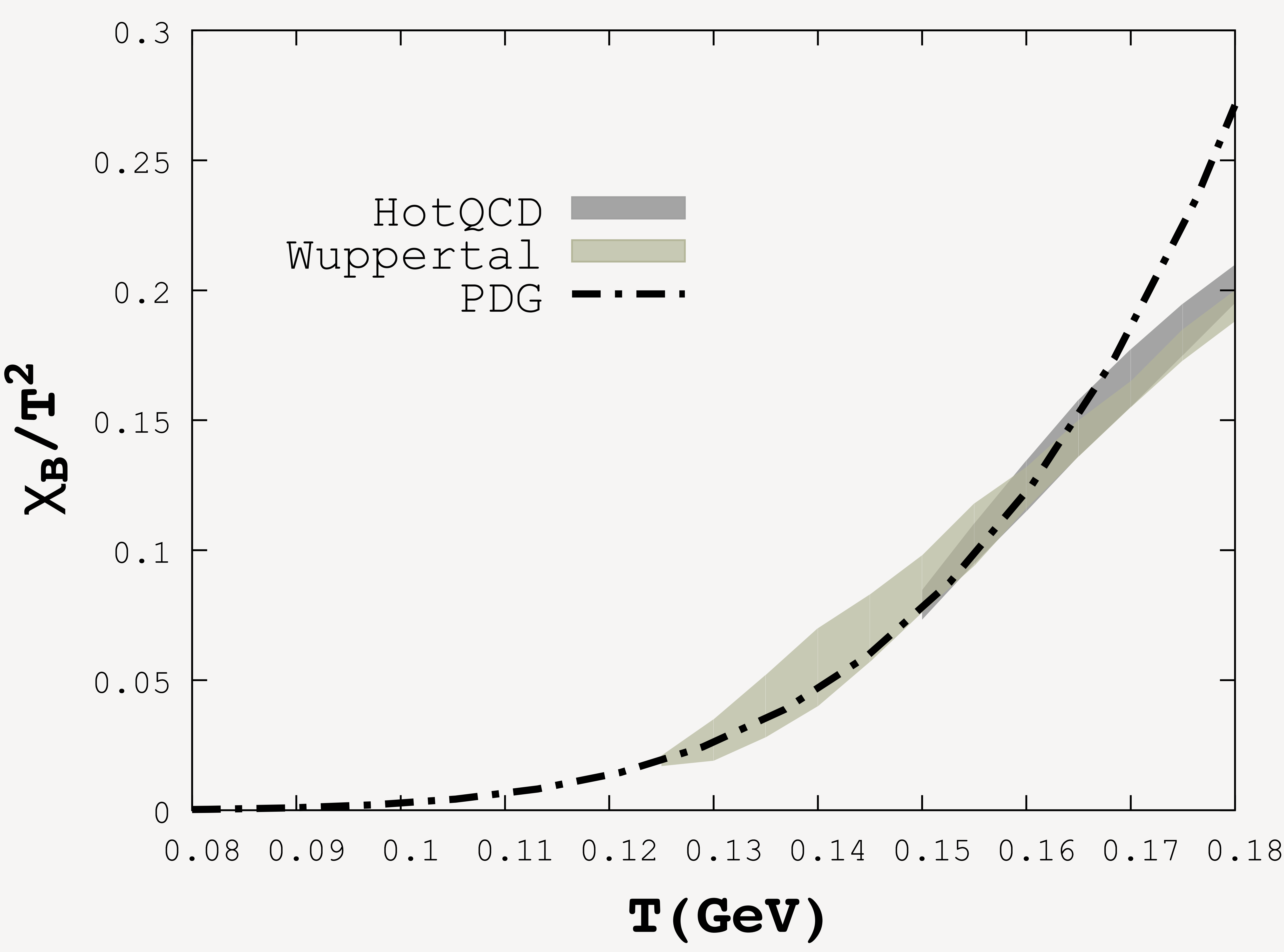
- taking derivative

$$\chi_B = \frac{\partial^2}{\partial\bar{\mu}_B \partial\bar{\mu}_B} P \quad \textit{at the limit} \quad \mu_B \to 0$$

$\boxed{\text{probes fluctuations}}$

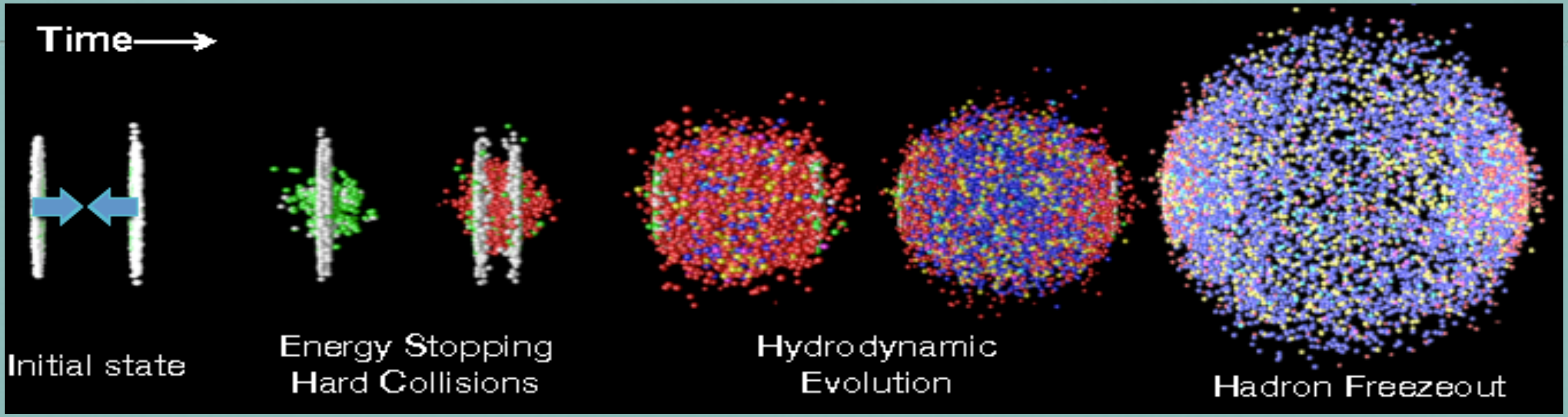
$$\chi_B = \frac{1}{\beta V} \frac{\partial^2}{\partial\bar{\mu}_B \partial\bar{\mu}_B} \ln Z$$

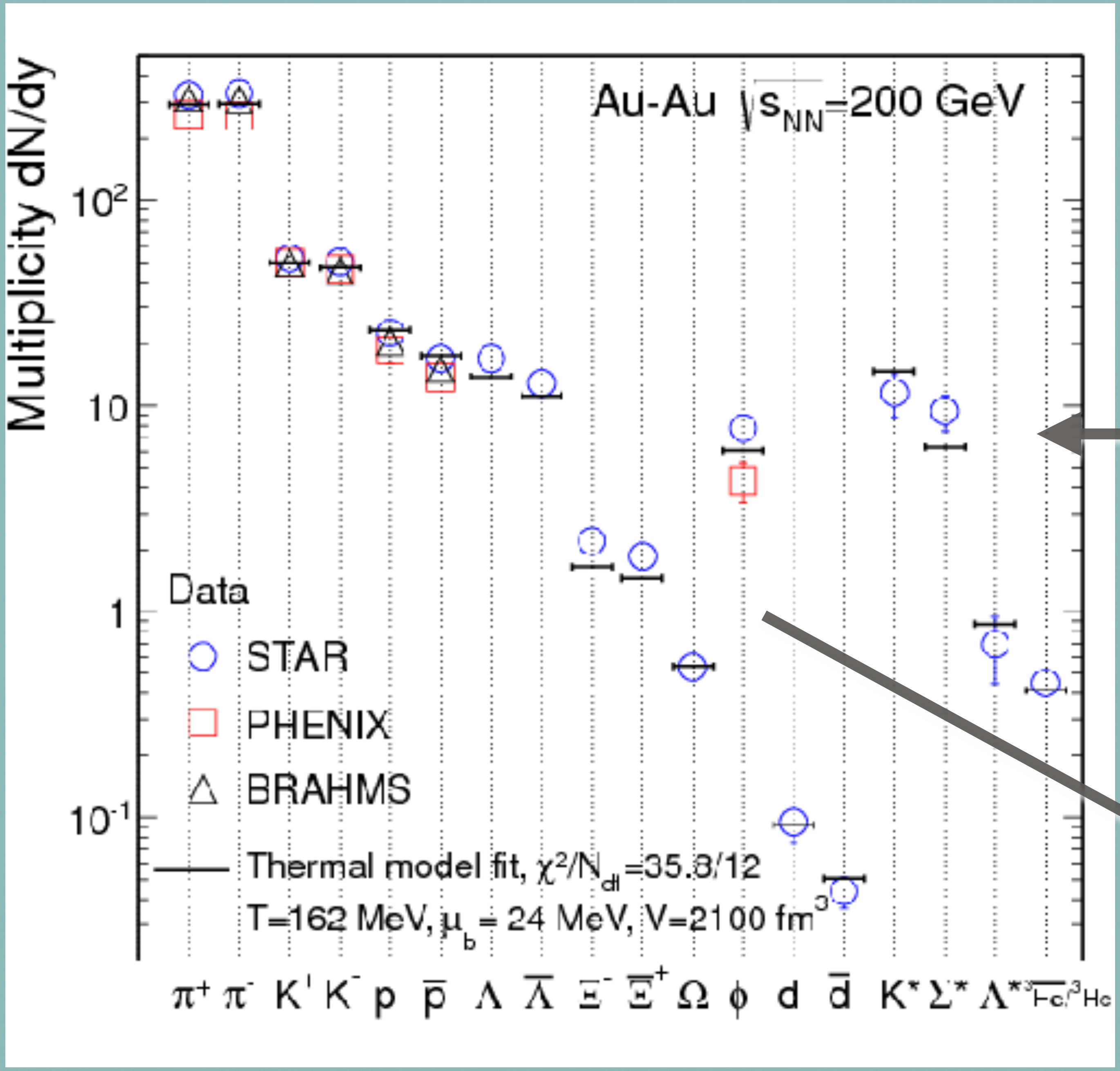
$$= T^2 \langle\langle \int d^4x \, \bar{\psi}(x)\gamma^0\psi(x)\bar{\psi}(0)\gamma^0\psi(0)\rangle\rangle_c$$



HotQCD
Wuppertal
PDG
χ_B/T^2
T(GeV)
0
0.05
0.1
0.15
0.2
0.25
0.3
0.08
0.09
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18

Central Au+Au 200 GeV/nucleon
MADAI
Simulation with UrQMD





freezeout hadrons yields described by HRG

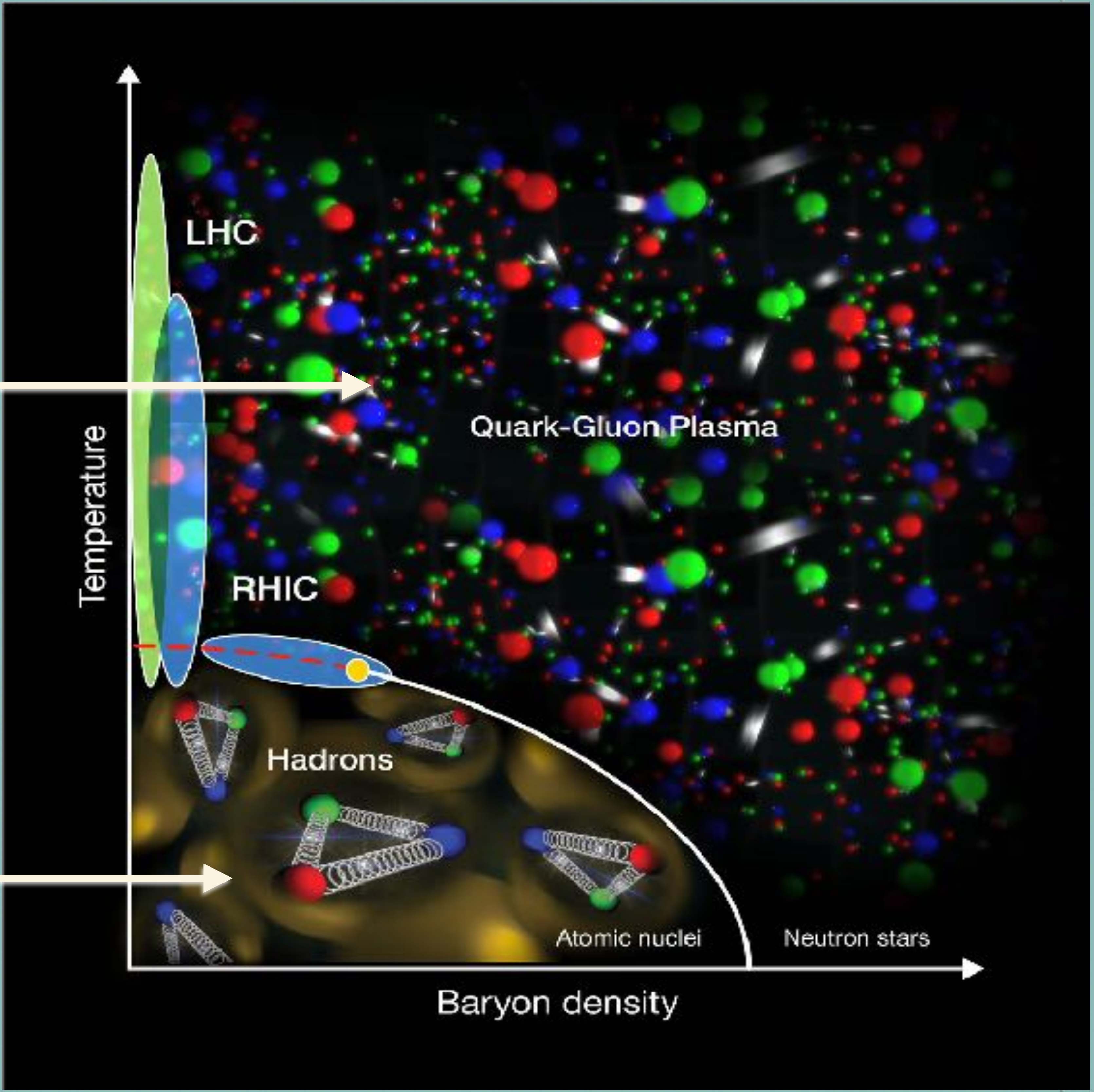
Freezeout parameters

$$T^f, \mu_B^f, \mu_S^f, \mu_Q^f, \ldots$$

Andronic, A. *et al.* Nucl.Phys. A904-905 (2013)

QCD Phase Diagram

QGP:
quarks and gluons
are deconfined.



Hadronic phase:
quarks are confined
and massive.

Courtesy of Brookhaven National Laboratory

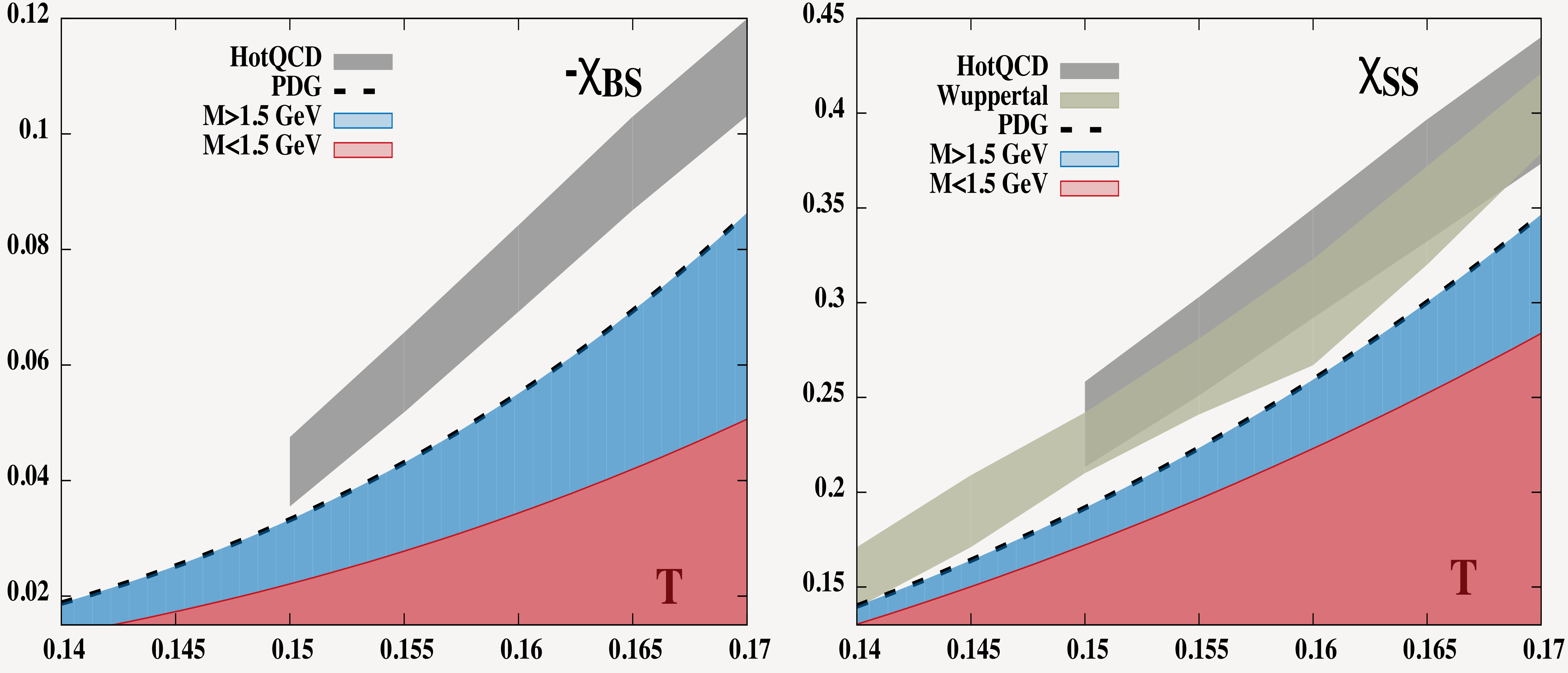
TOWARDS REAL HADRON GAS

- Hadron contents in individual sectors

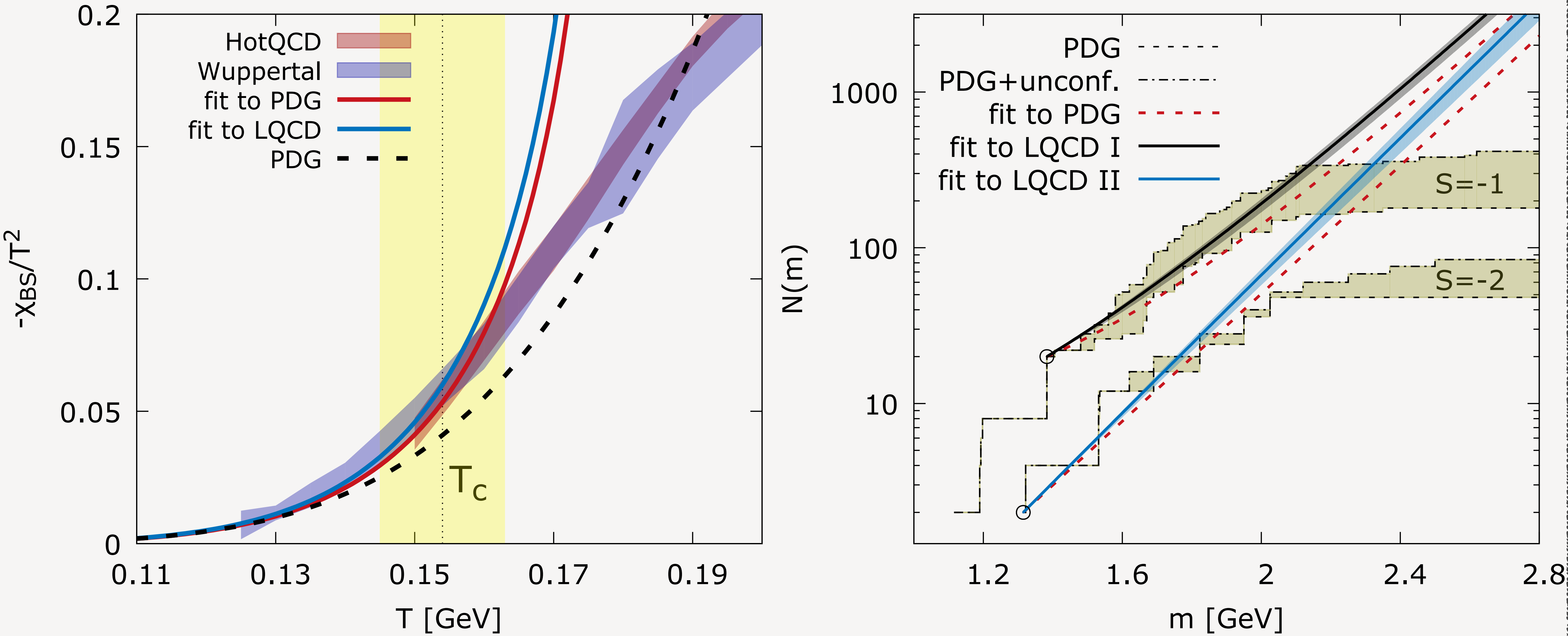
  -> the case of missing strange baryons

- Question the assumption of HRG treatment for resonances: non-interacting and point-like.

Missing resonances in the strange sector



strange mesons to be discovered...



PML, M. Marczenko, K. Redlich and C. Sasaki
Phys.Rev. C92 (2015) no.5, 055206

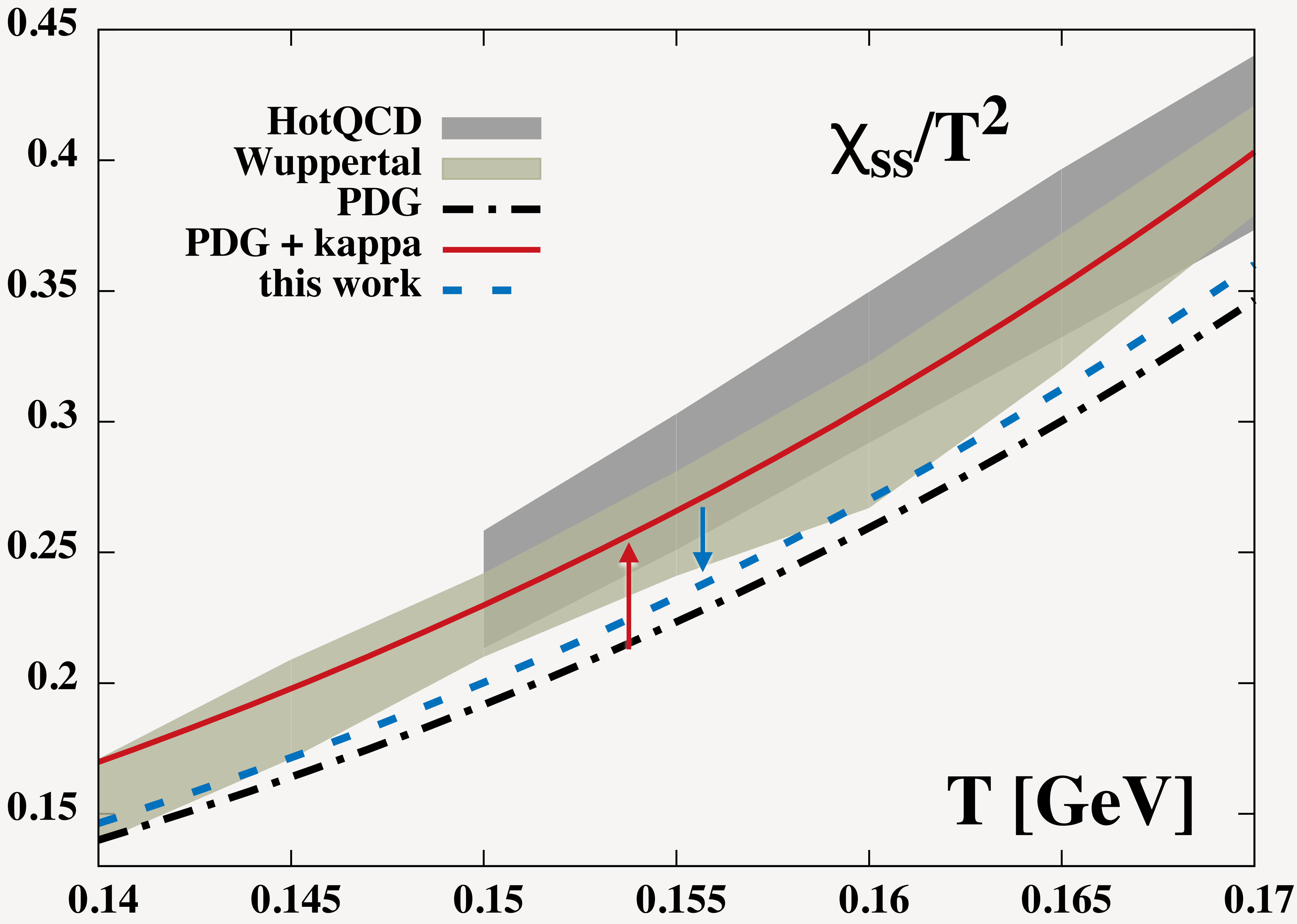
THERMODYNAMICS OF BROAD RESONANCES

- unconfirmed light resonances in the strange sector

$\boxed{K_0^*(800) \text{ or } \kappa}$ $\qquad I(J^P) = \frac{1}{2}(0^+)$

OMITTED FROM SUMMARY TABLE

Needs confirmation. See the mini-review on scalar mesons under $f_0(500)$ (see the index for the page number).



HotQCD
Wuppertal
PDG
PDG + kappa
this work
χ_{ss}/T^2
T [GeV]
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.14
0.145
0.15
0.155
0.16
0.165
0.17

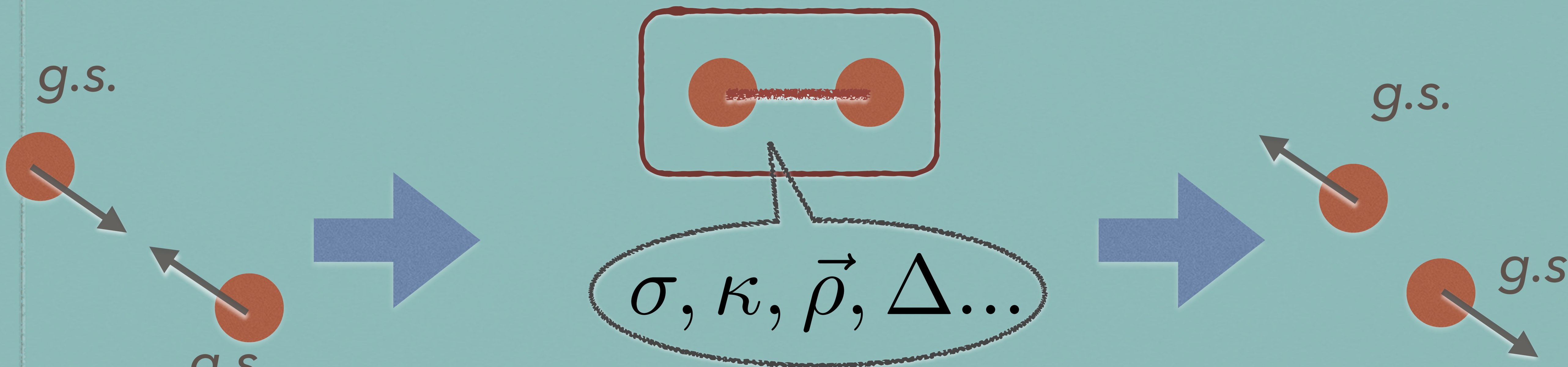
THERMODYNAMICS OF BROAD RESONANCES

- The κ meson has the right mass range.
- But it also has a broad width!

WHAT IS THE EFFECT OF RESONANCE'S WIDTH ON THERMODYNAMICS?

S-MATRIX APPROACH

S-MATRIX APPROACH

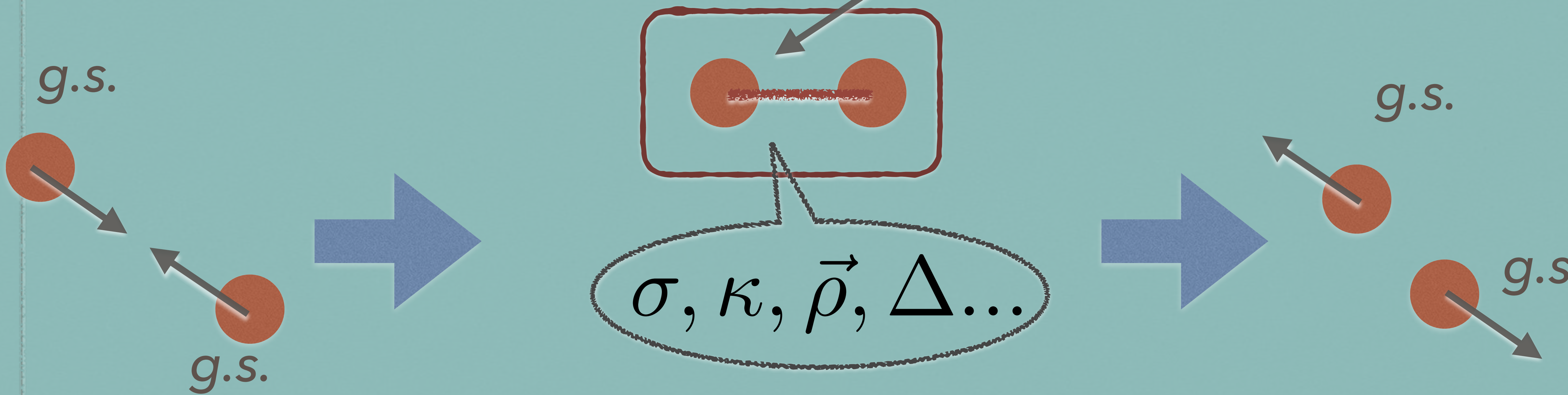


+ repulsive forces

consistent treatment of both
attractive and repulsive forces

S-MATRIX APPROACH

$$\rho_E \sim 2\frac{d\delta}{dE}$$



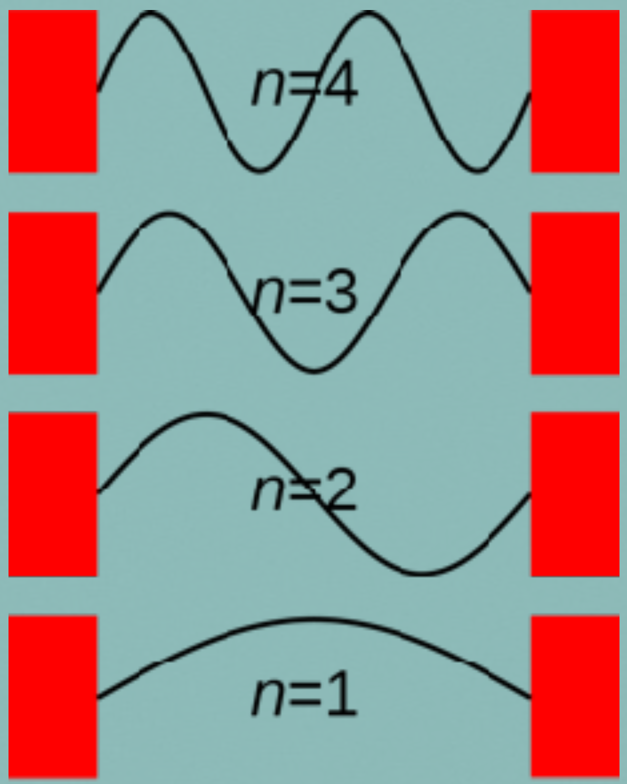
+ *repulsive forces*

consistent treatment of both attractive and repulsive forces

PHASE SHIFT AND DENSITY OF STATES

particle in a box

$$\psi \sim \sin(k^{(0)}x) \qquad k^{(0)} = \frac{n\pi}{L}$$



PHASE SHIFT AND DENSITY OF STATES

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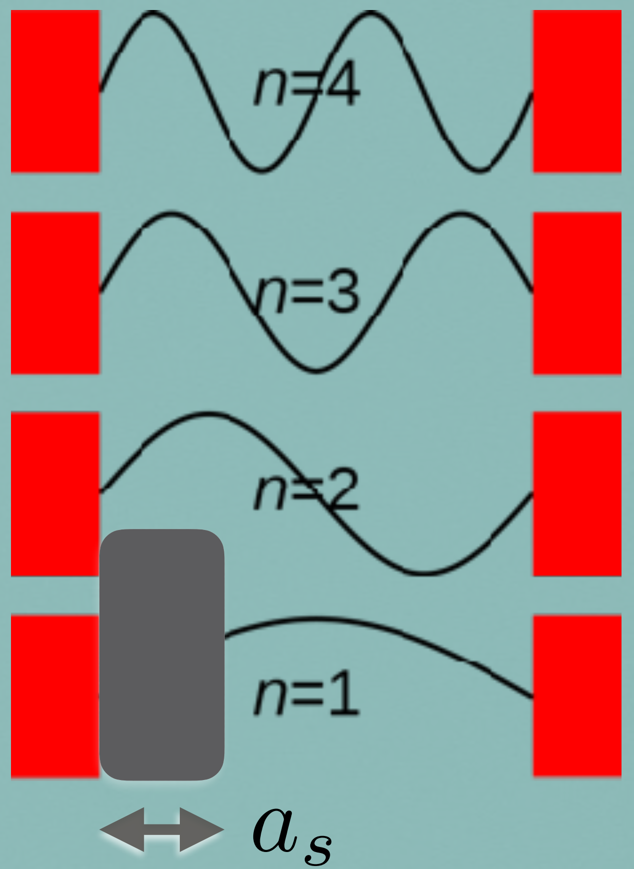


in the presence of a scattering potential

$$\psi \sim \sin(kx + \delta(k))$$

density of states

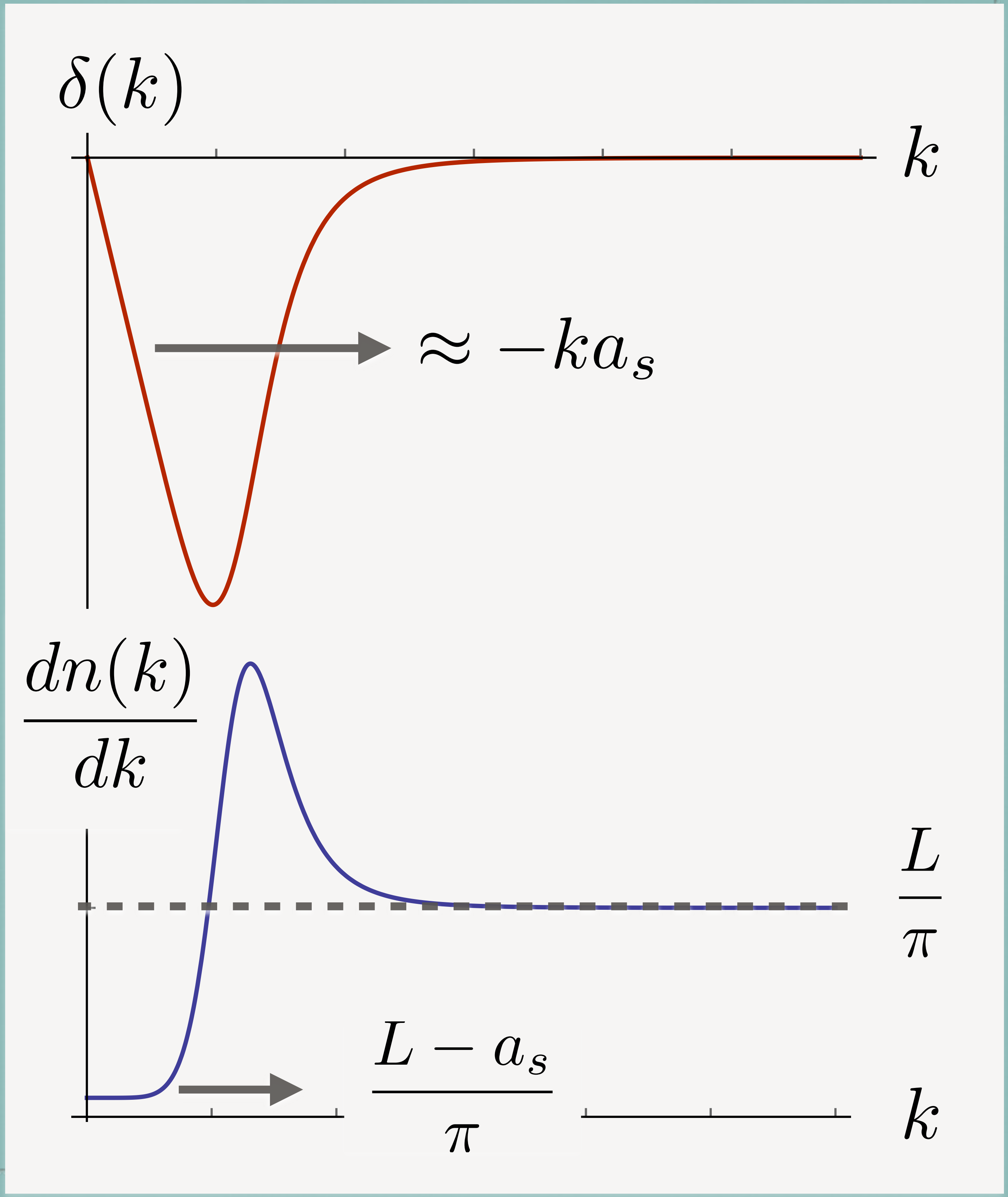
$$kL + \delta(k) = n\pi \quad \Rightarrow \quad \frac{dn(k)}{dk} = \frac{L}{\pi} + \frac{1}{\pi}\frac{d\delta}{dk}$$

PHASE SHIFT AND DENSITY OF STATES

$$\frac{dn(k)}{dk} = \frac{L}{\pi} + \boxed{\frac{1}{\pi}\frac{d\delta}{dk}}$$

change in d.o.s. due to int.

Effect of repulsive interaction: pushing states from low k to high k



phase shift and d.o.s. (schematics)

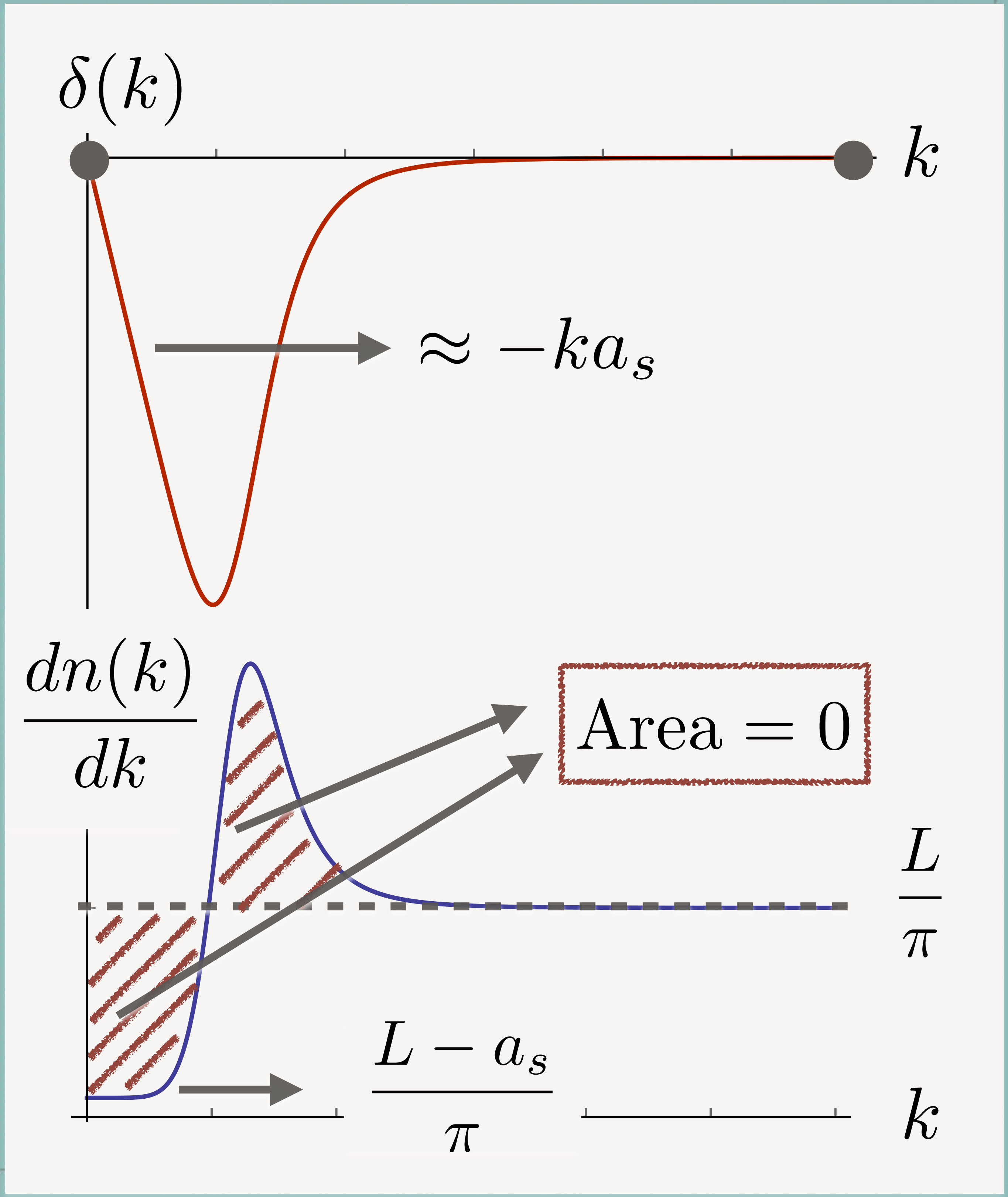
PHASE SHIFT AND DENSITY OF STATES

$$\frac{dn(k)}{dk} = \frac{L}{\pi} + \boxed{\frac{1}{\pi}\frac{d\delta}{dk}}$$

sum rule
(Levinson's theorem)

$$\int_0^\infty dk\, \frac{1}{\pi}\delta' = \frac{\delta(\infty) - \delta(0)}{\pi}$$

n_{int}



phase shift and d.o.s. (schematics)

S-MATRIX FORMULATION OF THERMODYNAMICS

$$\Delta \ln Z = \int dE\, e^{-\beta E} \frac{1}{4\pi i} \operatorname{tr} \left\{ S_E^{-1} \overleftrightarrow{\frac{\partial}{\partial E}} S_E \right\}_c$$

R. Dashen, S. K. Ma and H. J. Bernstein,
Phys. Rev. 187 (1969) 345.

A SIMPLE TRICK

$$S_E = e^{2i\delta_E}$$

$$\frac{1}{4\pi i}\,\mathrm{tr}\left\{S_E^{-1}\frac{\overleftrightarrow{\partial}}{\partial E}S_E\right\}_c$$

$$= \frac{1}{2\pi} \times 2\,\frac{\partial}{\partial E}\,\boxed{\frac{1}{2}\,\mathrm{Im}\,\mathrm{tr}\left\{\ln S_E\right\}}$$

$$\Delta \ln Z = \int dE\, e^{-\beta E} \times \frac{1}{\pi}\frac{\partial}{\partial E}\,\mathrm{tr}\,\left(\delta_E\right).$$

E. Beth and G. Uhlenbeck,
Physica (Amsterdam) 4, 915 (1937).

A SIMPLE TRICK

$$\frac{1}{4\pi i}\,\mathrm{tr}\left\{S_E^{-1}\frac{\overleftrightarrow{\partial}}{\partial E}S_E\right\}_c$$

$$S_E = e^{2i\delta_E}$$

$$= \frac{1}{2\pi}\times 2\,\frac{\partial}{\partial E}\,\boxed{\frac{1}{2}\,\mathrm{Im}\,\mathrm{tr}\left\{\ln S_E\right\}}$$

$$\mathcal{Q}(E) \longrightarrow$$ *Generalised phase shift*

$$B = 2\frac{\partial}{\partial E}\mathcal{Q}(E) \longrightarrow$$ *Generalised spectral function*

EXERCISE: QM SCATTERING OPERATOR

show that

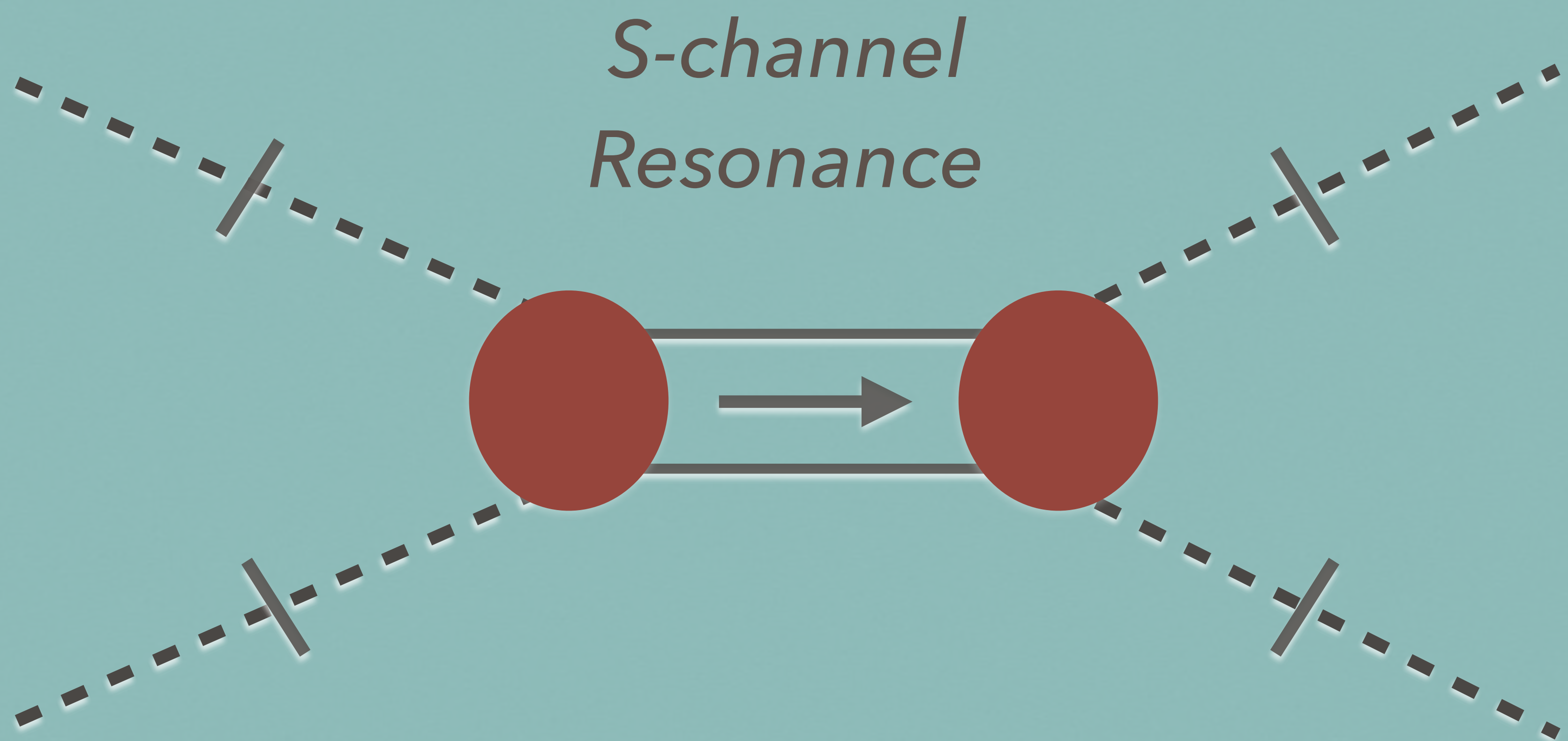
$$S_E = G_0^* \, G^{*\,-1} \, G \, G_0^{-1}$$
$$\quad = 1 - 2\pi i \times \delta(E - H_0) \times T_E$$

$$G = \frac{1}{E - H + i\epsilon}$$

Verify $\Delta \ln Z = \int dE \, e^{-\beta E} \frac{1}{4\pi i} \, \mathrm{tr} \left\{ S_E^{-1} \frac{\overleftrightarrow{\partial}}{\partial E} S_E \right\}_c$

Alternative way to obtain the Beth-Uhlenbeck result!

ILLUSTRATION: S-MATRIX FOR RELATIVISTIC RESONANCES



$$i\mathcal{M}_E \approx (-ig)\,\frac{i}{E^2 - m_{\rm res}^2 + iE\,\gamma_E}\,(-ig)$$

$$\begin{aligned}\mathcal{Q}(E) &= \frac{1}{2}\,\mathrm{Im}\,\mathrm{tr}\,\{\ln S_E\} \\ &= \frac{1}{2}\,\mathrm{Im}\,\ln\left[1 + \int d\phi_2\, i\mathcal{M}_E\right]\end{aligned}$$

$$\int d\phi_2\, i\mathcal{M}_E = \frac{-i\,2\,E\,\gamma_E}{E^2 - m_{\rm res}^2 + iE\,\gamma_E}$$

$$= 2\,i\,\sin\delta_E\, e^{i\delta_E}$$

with

$$\tan\delta_E = \frac{-E\,\gamma_E}{E^2 - m_{\rm res}^2}$$

=> $\mathcal{Q}(E) = \delta_E$

HRG approx.

$$\delta_E = \pi \times \theta(E - m_{\rm res})$$

FORMULATION

given the exact phase shift δ_l

from theory
or
from experiment

thermodynamics

$$B_l = 2\frac{d}{dq}\delta_l$$

eff. spectral function

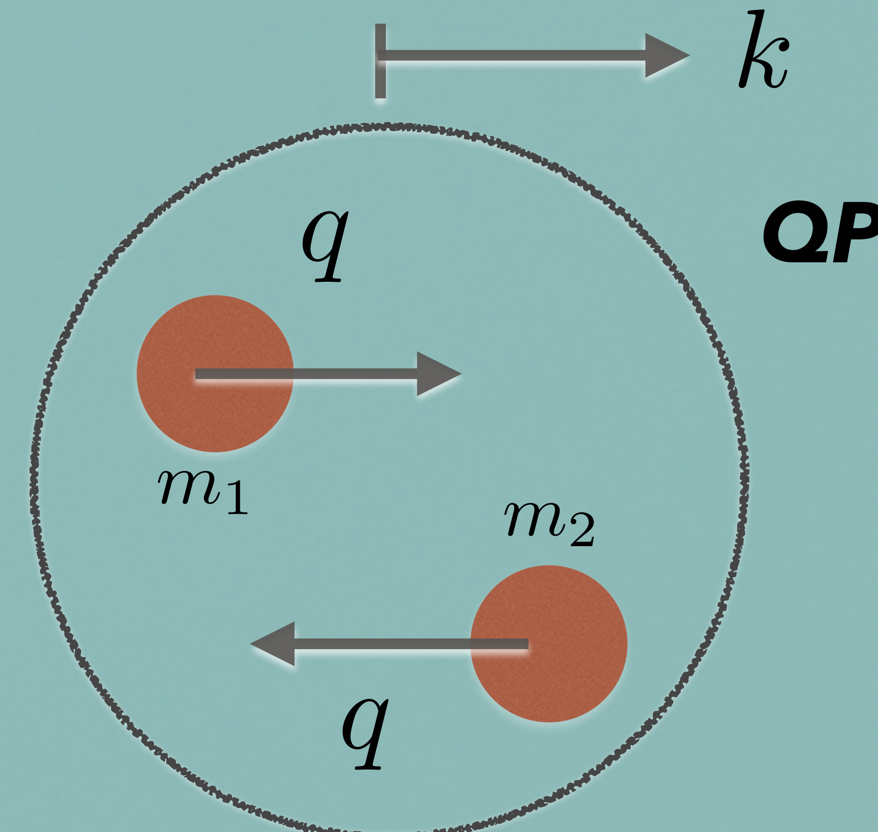
$$P = P^{(0)} + \Delta P^{B.U.}$$

free gas + interaction

FORMULATION

dynamical ***statistical (thermal weight)***

$$\Delta P^{\mathrm{B.U.}} = (2l+1)\int \frac{dq}{2\pi} B_l(q) \int \frac{d^3k}{(2\pi)^3} T \ln\left(1 + e^{-\beta E(k,q,m_i)}\right)$$



$$E = \sqrt{k^2 + M(q)^2}$$

$$M(q) = \sqrt{q^2 + m_1^2} + \sqrt{q^2 + m_2^2}$$

$$M(q) = \sqrt{q^2 + m_1^2} + \sqrt{q^2 + m_2^2}$$

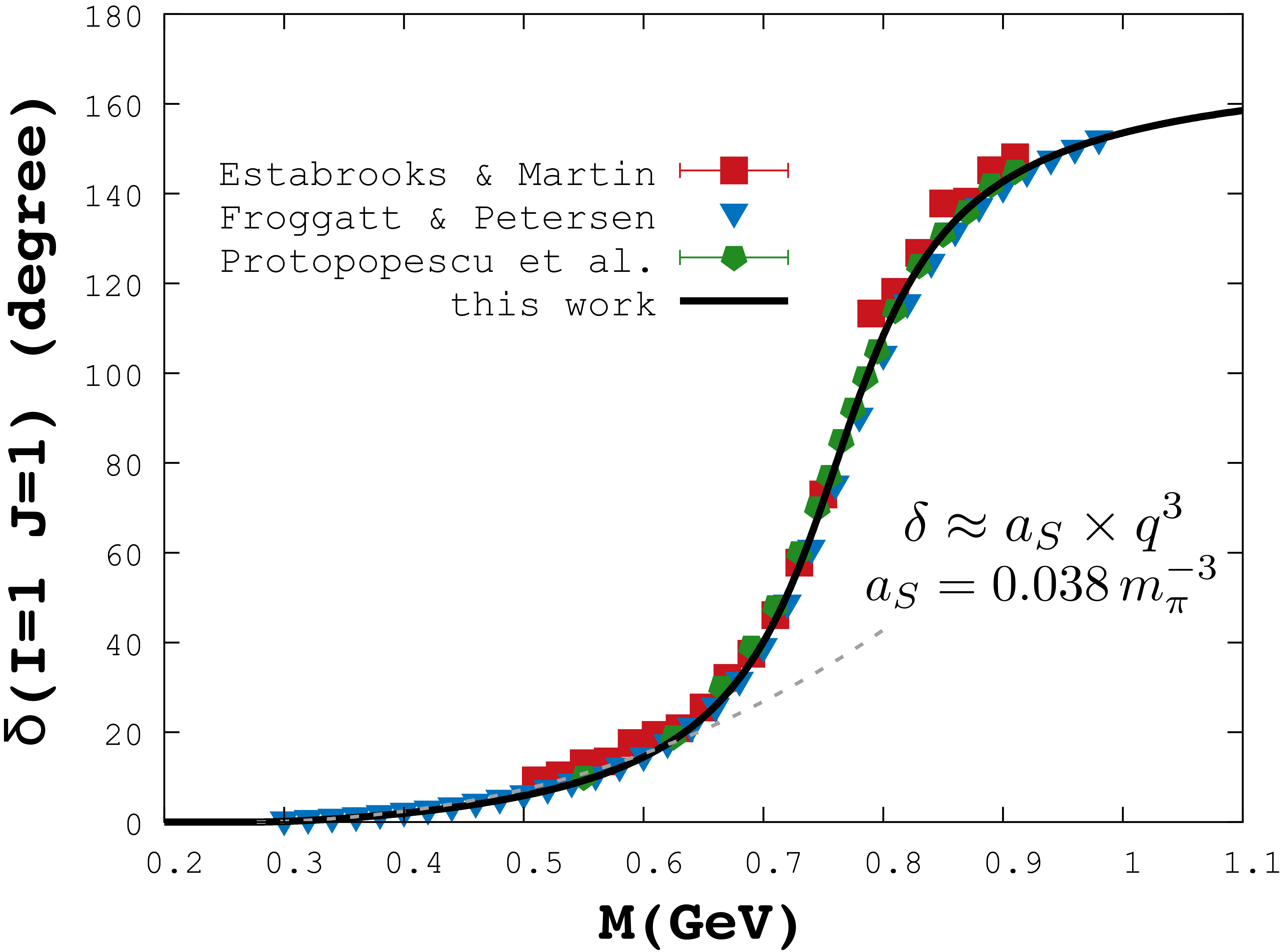
$$B_l = 2\frac{d}{dq}\delta_l$$

WHAT'S IN A NAME? THAT WHICH WE CALL A RESONANCES?

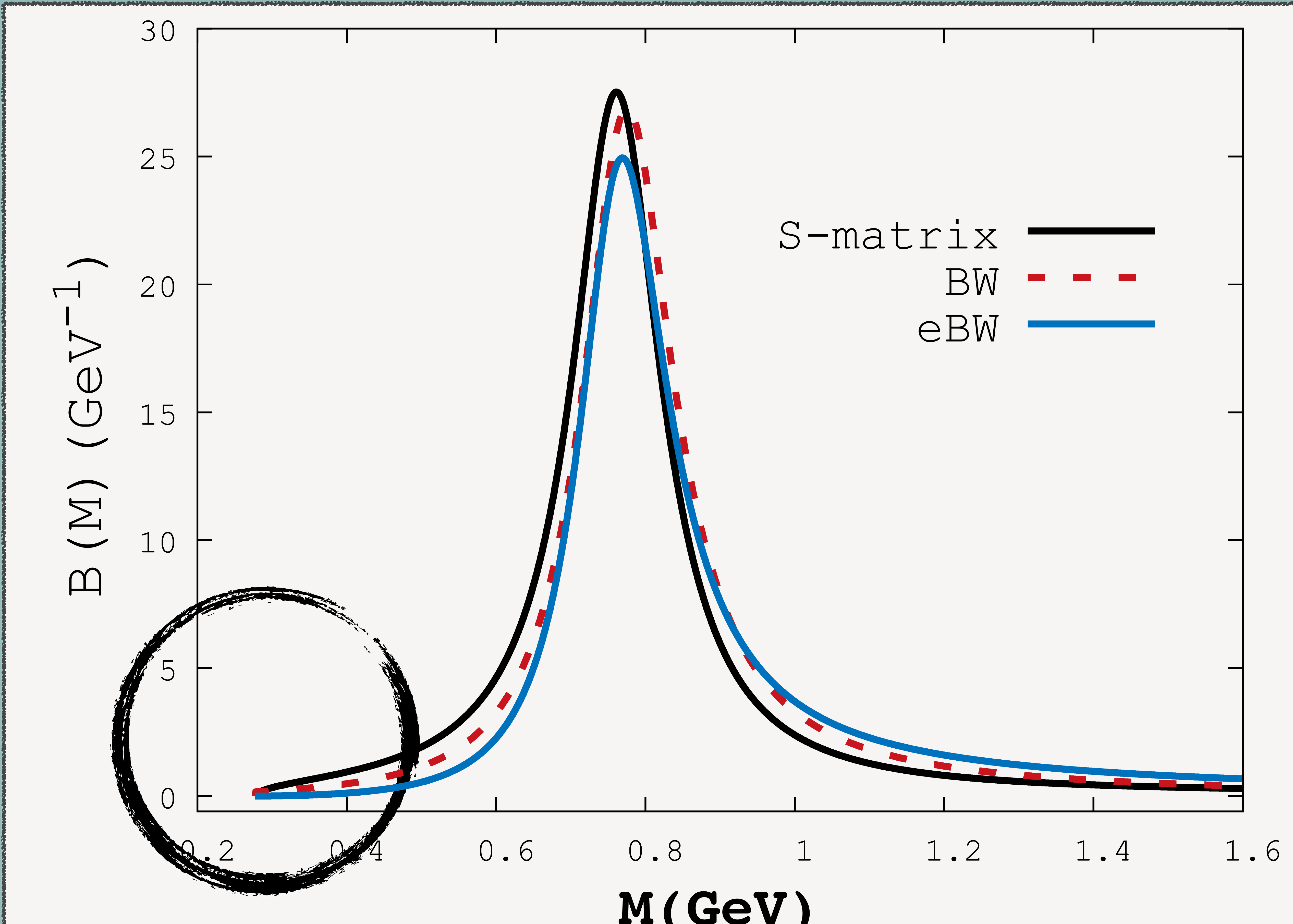
- A resonance is MORE than a MASS and a **WIDTH**

$\boxed{\boldsymbol{\rho(770)}\ ^{[h]}}$ $\qquad I^G(J^{PC}) = 1^+(1^{--})$

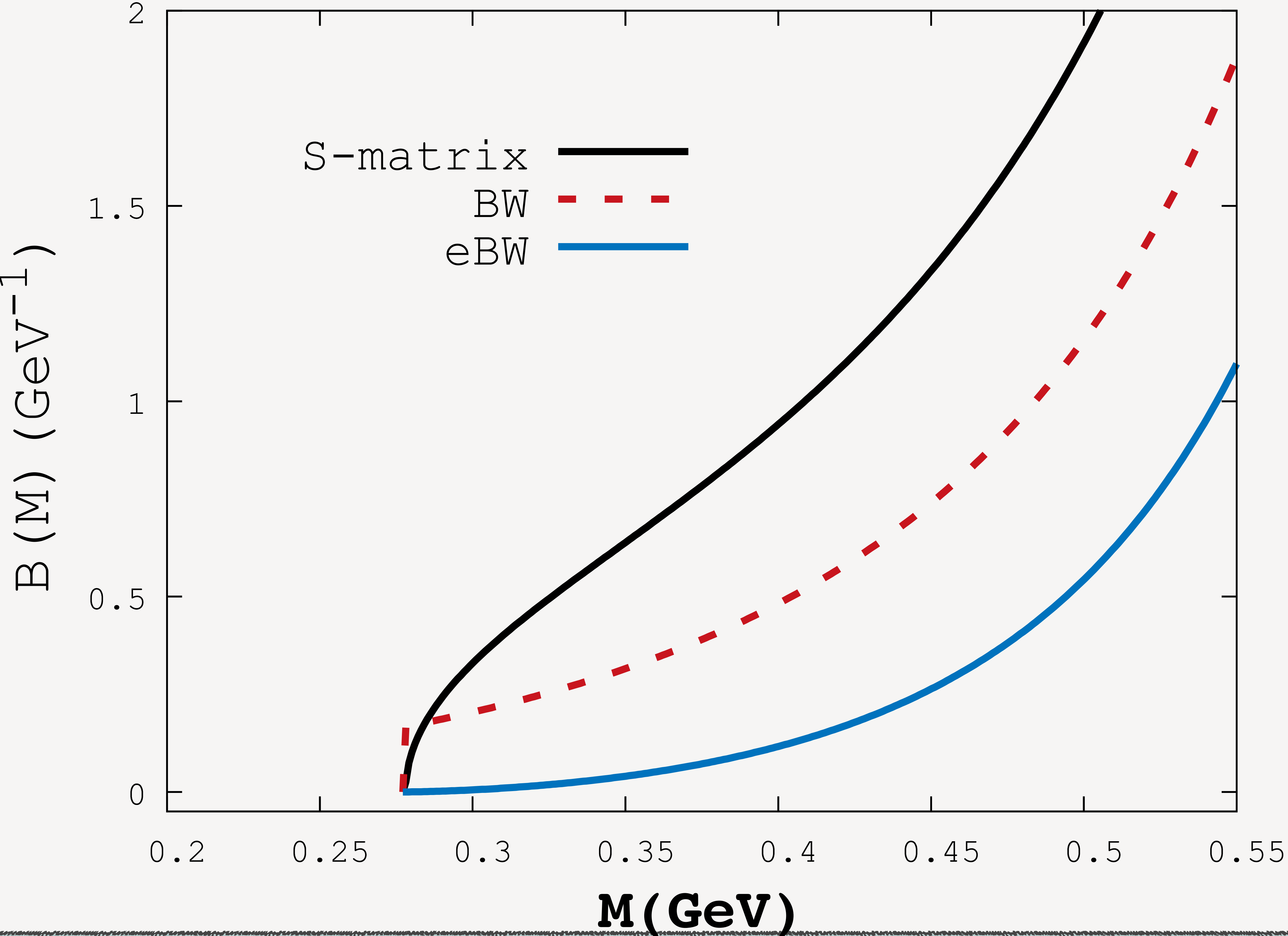
Mass $m = 775.26 \pm 0.25$ MeV
Full width $\Gamma = 149.1 \pm 0.8$ MeV
$\Gamma_{ee} = 7.04 \pm 0.06$ keV



δ(I=1 J=1) (degree)
M(GeV)
Estabrooks & Martin
Froggatt & Petersen
Protopopescu et al.
this work
$\delta \approx a_S \times q^3$
$a_S = 0.038\, m_\pi^{-3}$
0
20
40
60
80
100
120
140
160
180
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1



30
25
20
15
10
5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
B(M) (GeV^{-1})
M(GeV)
S-matrix
BW
eBW



S-matrix
BW
eBW
B(M) (GeV^{-1})
M(GeV)
0
0.5
1
1.5
2
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55

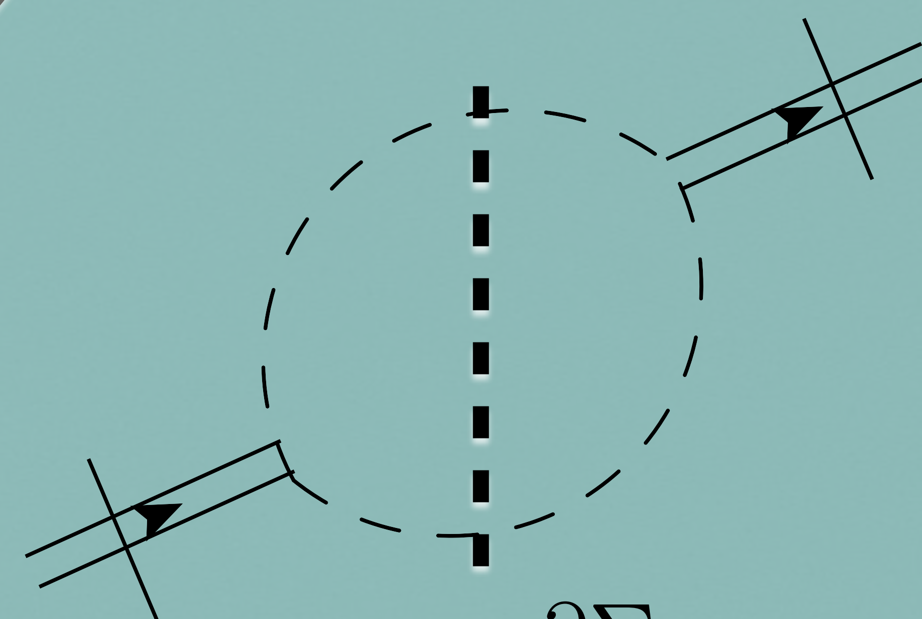
BETH-UHLENBECK APPROXIMATION

$$\delta = -\mathrm{Im}\,\mathrm{Tr}\ln G_\rho^{-1}$$

physical interpretation:

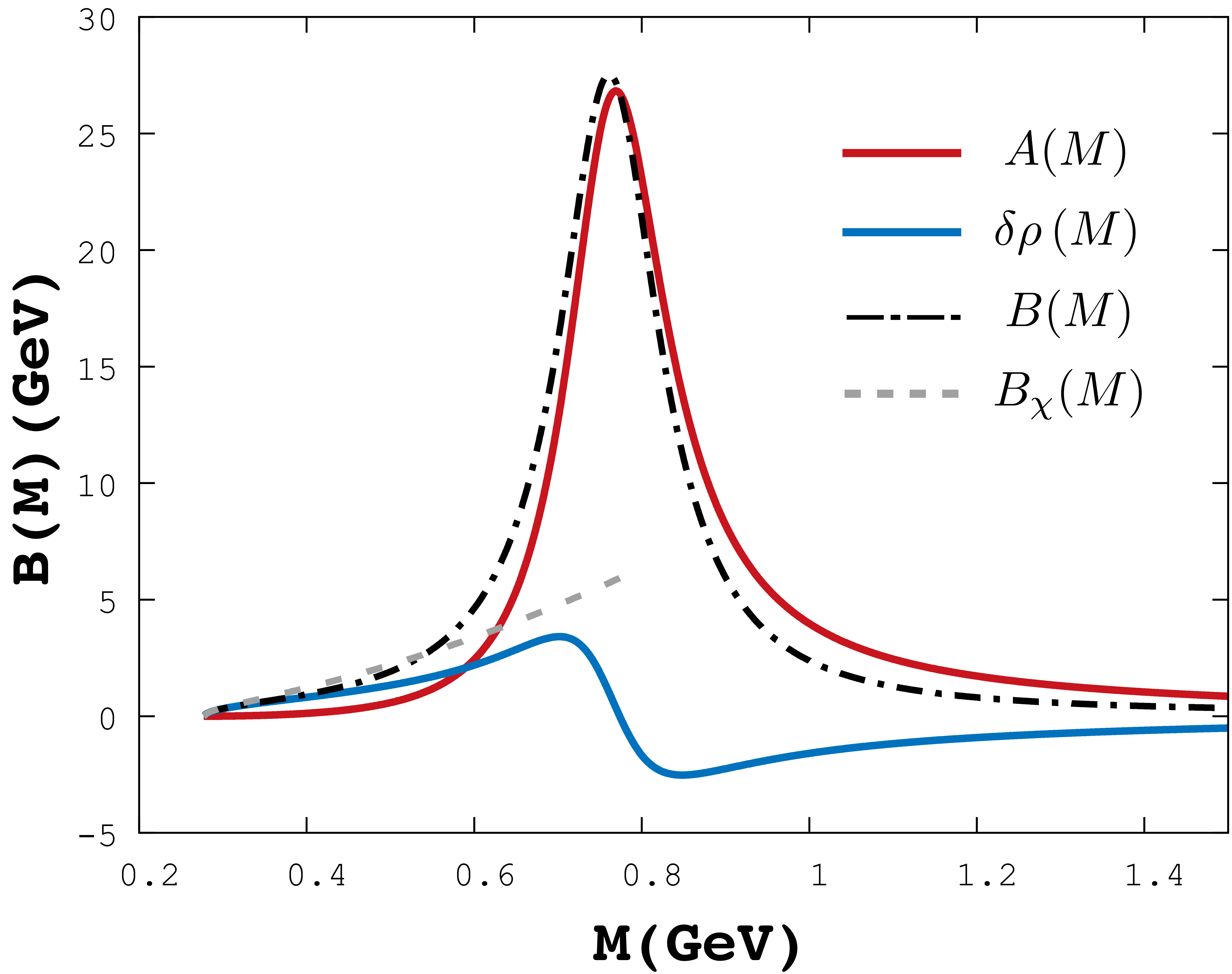
$$
\begin{aligned}
B &= 2\,\frac{\partial}{\partial E}\delta \\
&= -2\,\mathrm{Im}\frac{\partial}{\partial E}\ln G_\rho^{-1} \\
&= -2\,\mathrm{Im}[G_\rho](2E) + 2\,\mathrm{Im}[\frac{\partial \Sigma_\rho}{\partial E} G_\rho]
\end{aligned}
$$

contribution from correlated pi pi pair

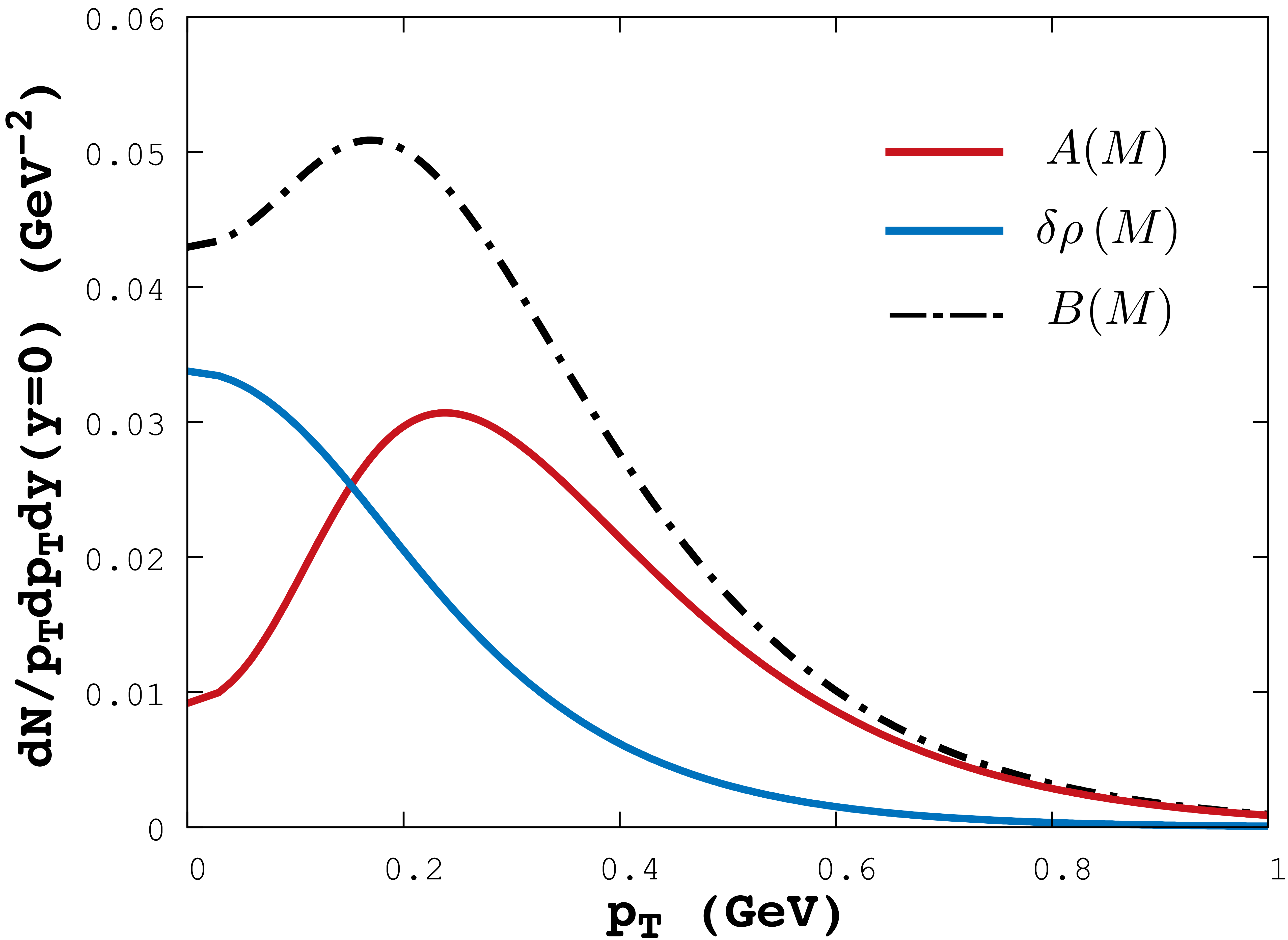


$$\frac{\partial \Sigma_\rho}{\partial E}$$

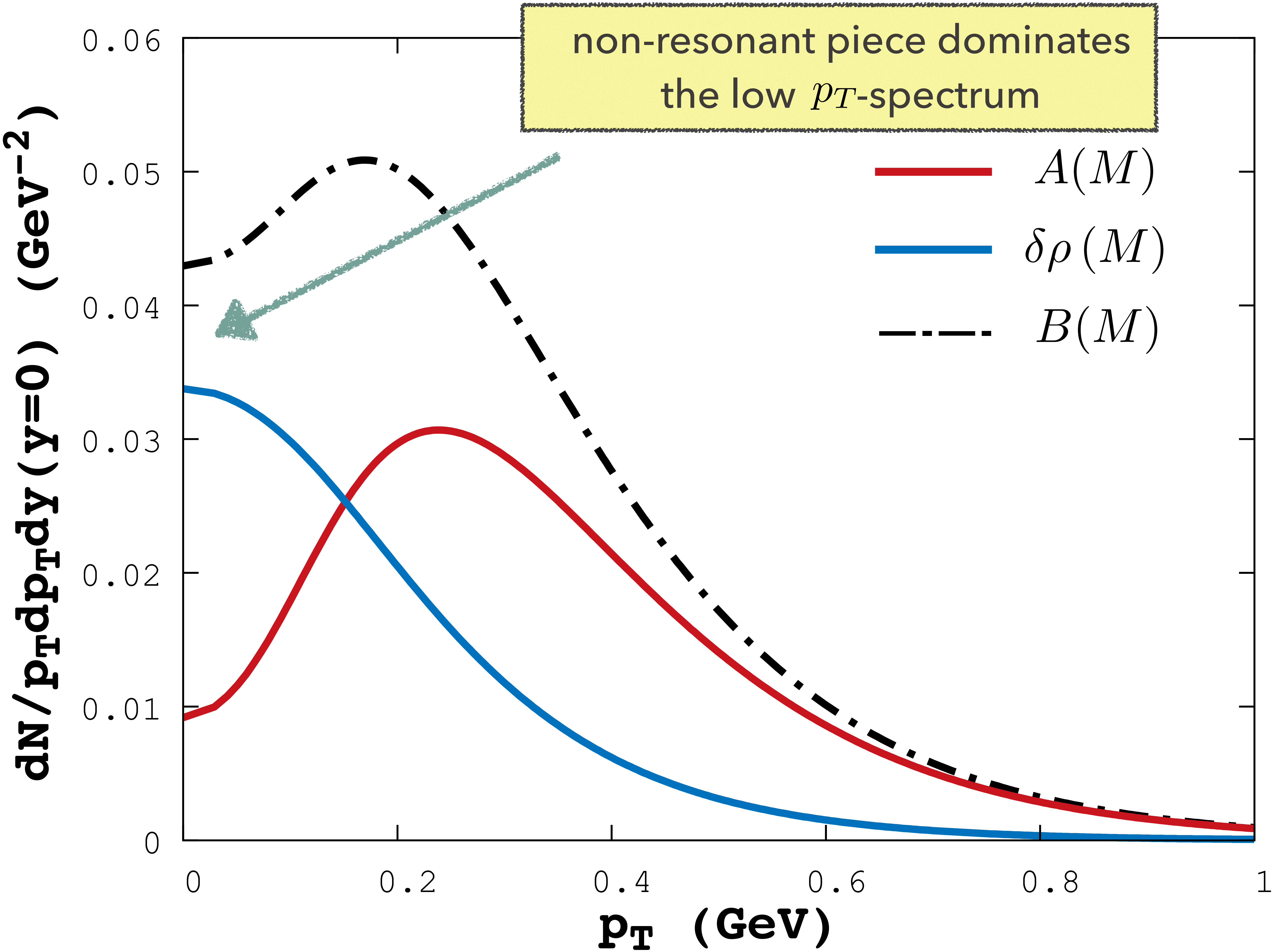
$$=> \boxed{\rho_\rho(E) + \delta\rho_\rho(E)}$$



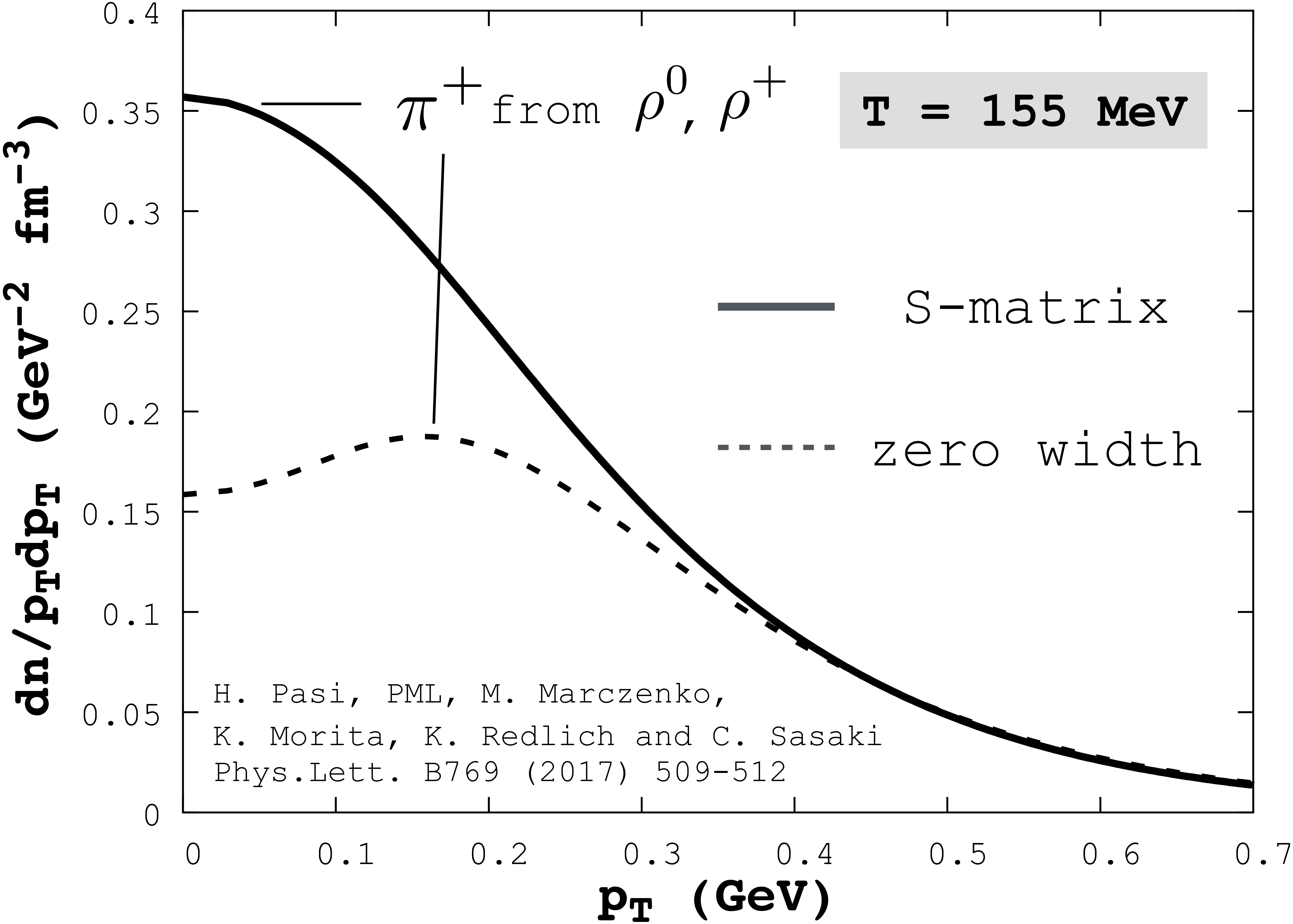
B(M) (GeV)
M(GeV)
$A(M)$
$\delta\rho(M)$
$B(M)$
$B_\chi(M)$
30
25
20
15
10
5
0
-5
0.2
0.4
0.6
0.8
1
1.2
1.4



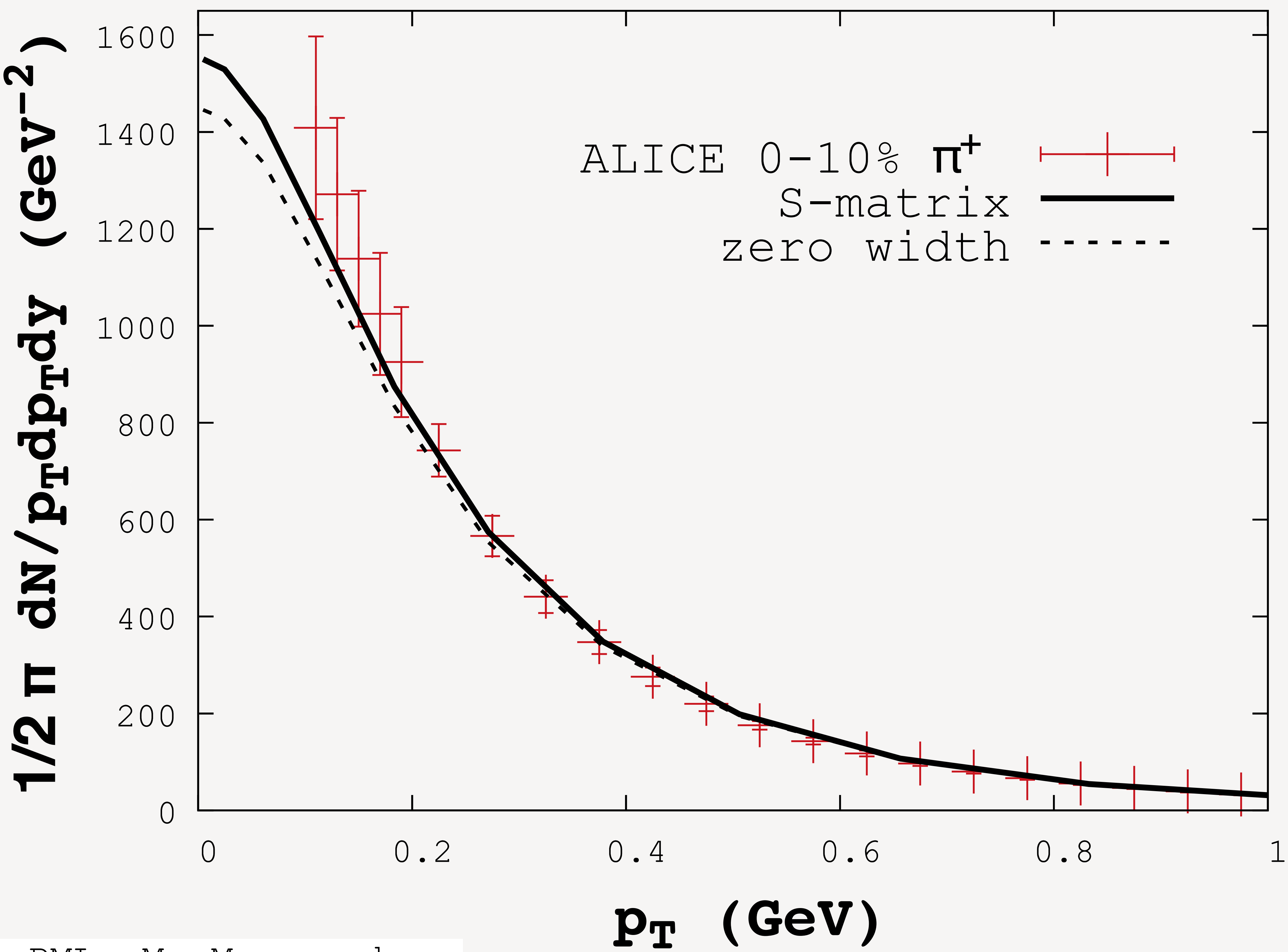
0.06
0.05
0.04
0.03
0.02
0.01
0
0
0.2
0.4
0.6
0.8
1
$A(M)$
$\delta\rho(M)$
$B(M)$
$dN/p_Tdp_Tdy(y=0)$ (GeV^{-2})
p_T (GeV)



non-resonant piece dominates
the low p_T-spectrum
$A(M)$
$\delta\rho(M)$
$B(M)$
0.06
0.05
0.04
0.03
0.02
0.01
0
0
0.2
0.4
0.6
0.8
1
$dN/p_T dp_T dy(y=0)$ (GeV^{-2})
p_T (GeV)

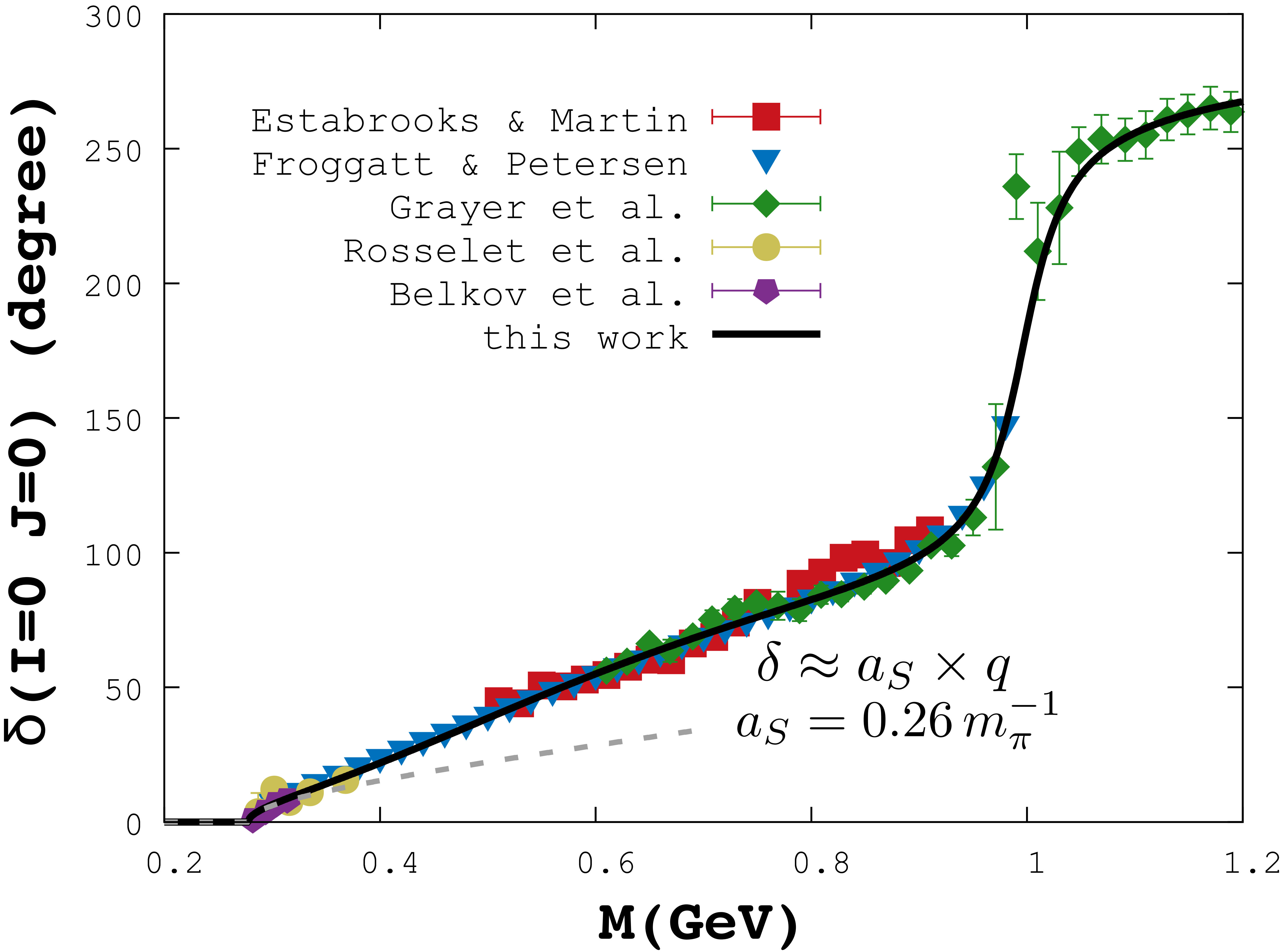


π^+ from ρ^0, ρ^+
T = 155 MeV
S-matrix
zero width
H. Pasi, PML, M. Marczenko,
K. Morita, K. Redlich and C. Sasaki
Phys.Lett. B769 (2017) 509-512
$dn/p_T dp_T$ (GeV^{-2} fm^{-3})
p_T (GeV)
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.5
0.6
0.7

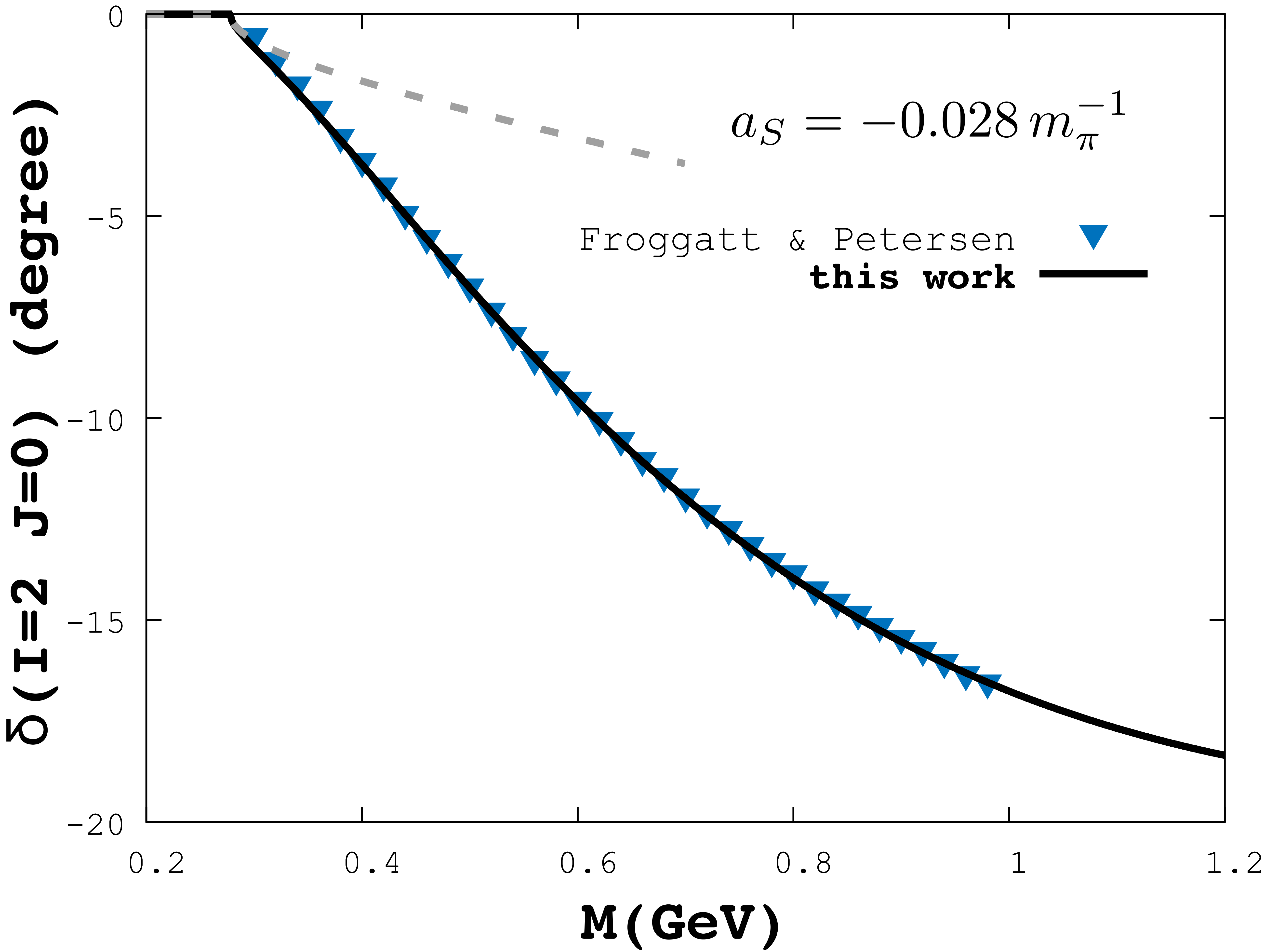


H. Pasi, PML, M. Marczenko,
K. Morita, K. Redlich and C. Sasaki
Phys.Lett. B769 (2017) 509-512

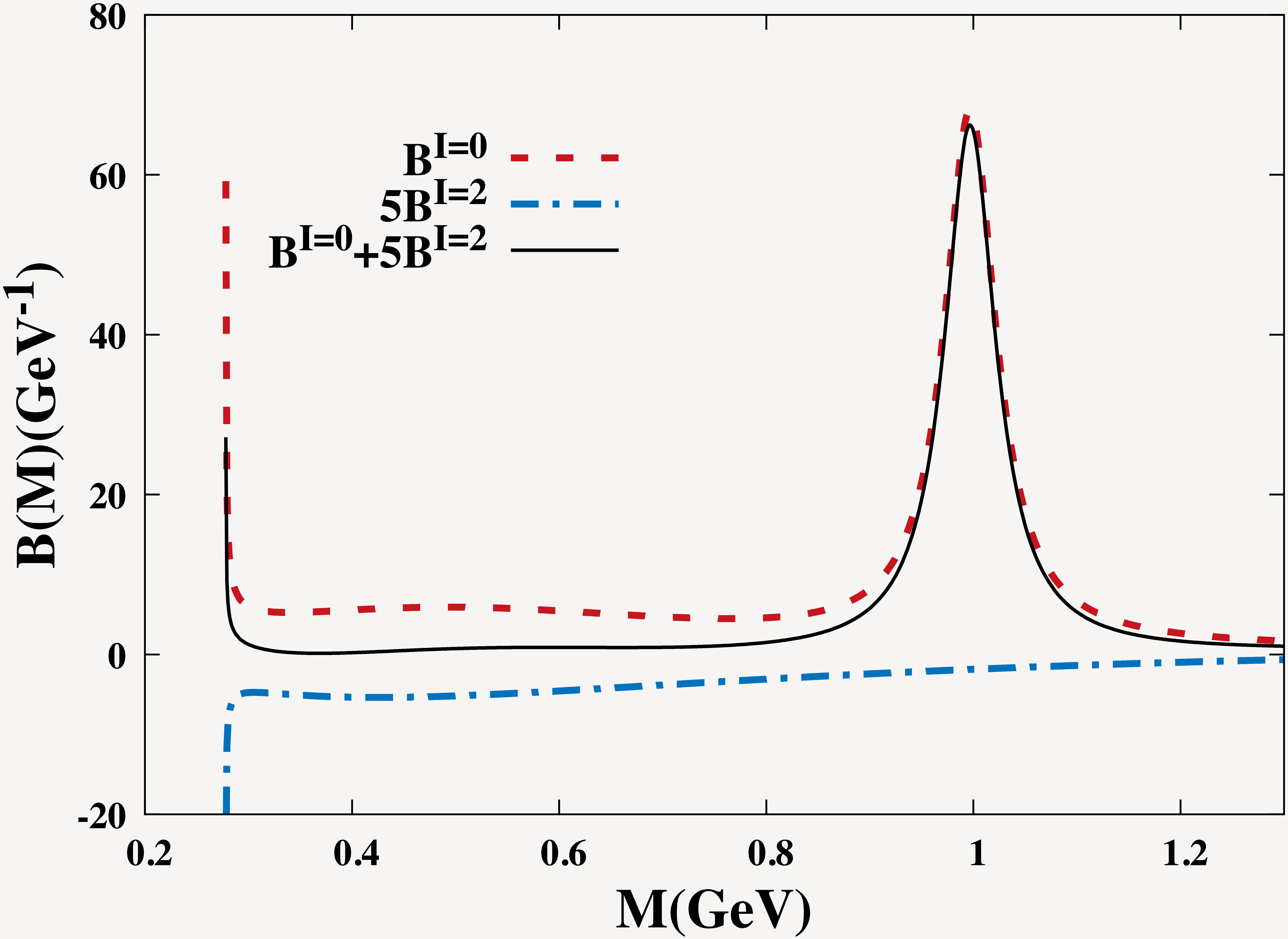
PI PI SCATTERING (S-WAVE)



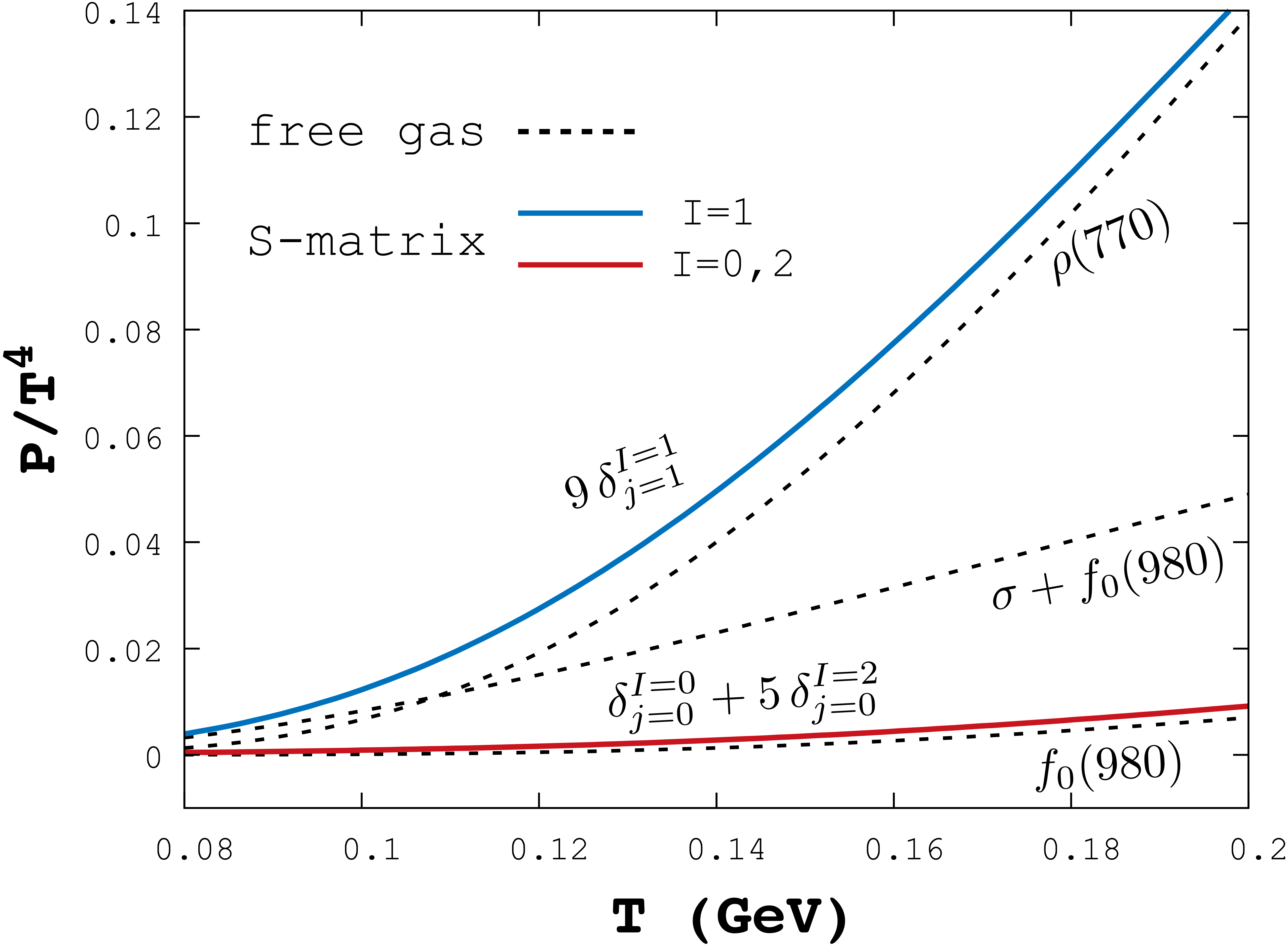
Estabrooks & Martin
Froggatt & Petersen
Grayer et al.
Rosselet et al.
Belkov et al.
this work
$\delta \approx a_S \times q$
$a_S = 0.26\, m_\pi^{-1}$
δ(I=0 J=0) (degree)
M(GeV)
0
50
100
150
200
250
300
0.2
0.4
0.6
0.8
1
1.2



$a_S = -0.028\, m_\pi^{-1}$
Froggatt & Petersen
this work
0
-5
-10
-15
-20
0.2
0.4
0.6
0.8
1
1.2
M(GeV)
δ(I=2 J=0) (degree)



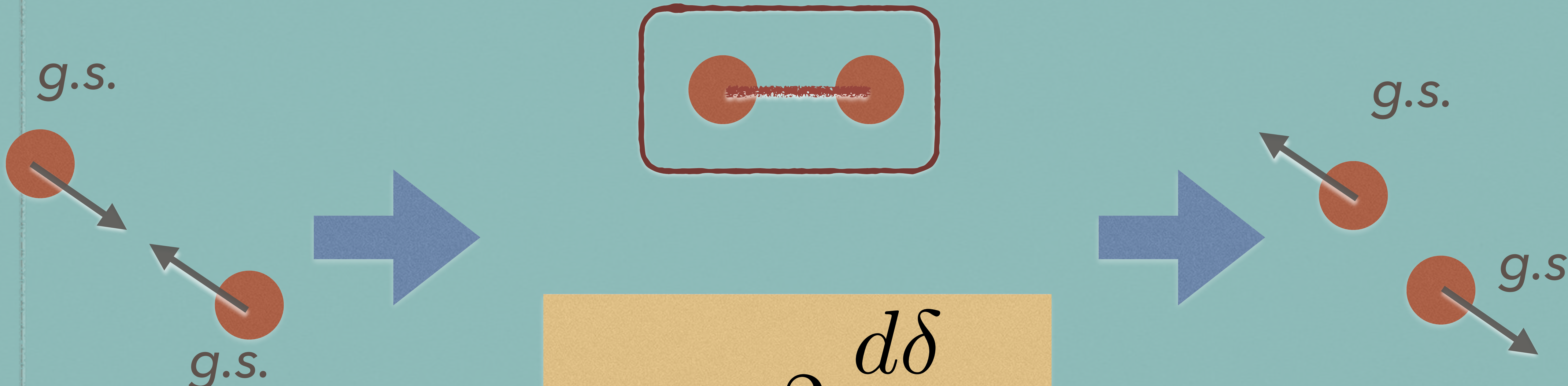
80
60
40
20
0
-20
0.2
0.4
0.6
0.8
1
1.2
M(GeV)
B(M)(GeV-1)
BI=0
5BI=2
BI=0+5BI=2



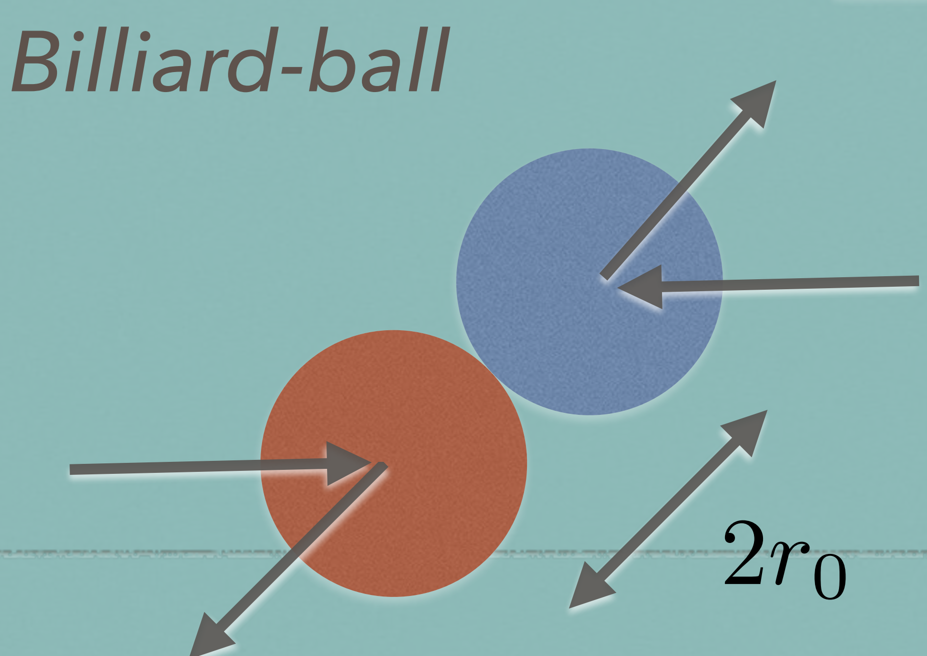
free gas
S-matrix
I=1
I=0,2
$\rho(770)$
$9\,\delta^{I=1}_{j=1}$
$\sigma + f_0(980)$
$\delta^{I=0}_{j=0} + 5\,\delta^{I=2}_{j=0}$
$f_0(980)$
P/T^4
T (GeV)
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0.08
0.1
0.12
0.14
0.16
0.18
0.2

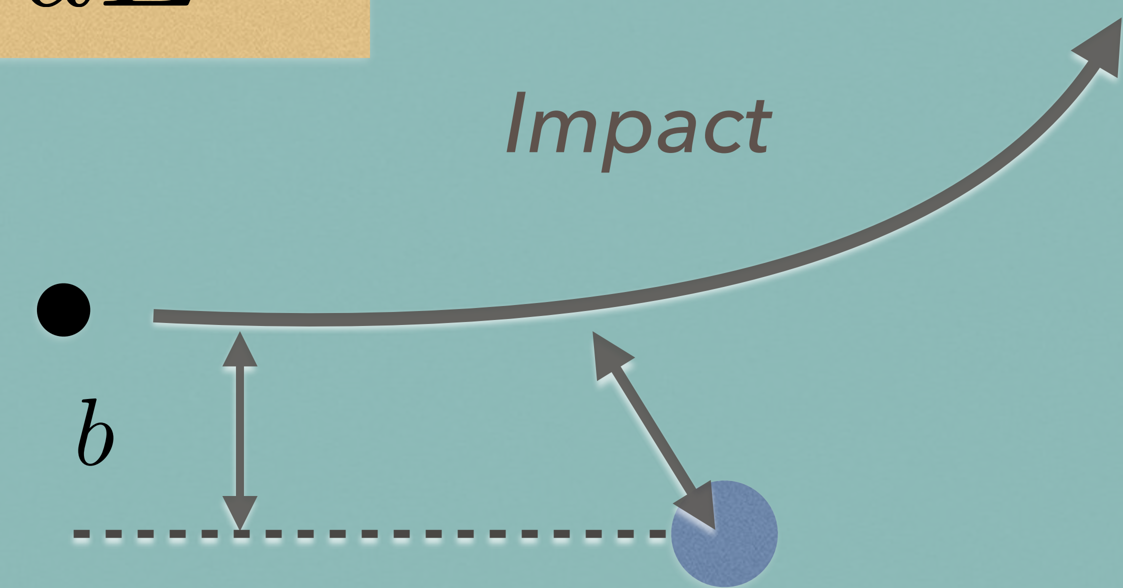
TIME DELAY

P. Danielewicz and S. Pratt
Phys.Rev. C53 (1996) 249-266



$$\tau \sim 2\frac{d\delta}{dE}$$





N-BODY SCATTERING

WHY N-BODY?

- EOS for dense system
  -> need higher coefficients of
  quantum cluster / virial expansion
  (three-body forces, etc.)
- Explore the influence of N-body scatterings on heavy ion collision observables:
  pT-spectra, flow etc.
- phenomenology
  -> model S-matrix element instead…

RECIPE

Feynman amplitude

- generalized phase shift

$$\mathcal{Q}_N(M) = \frac{1}{2}\,\mathrm{Im}\left[\ln\left(1 + \int d\phi_N\, i\mathcal{M}\right)\right]$$

$$d\phi_N = \frac{d^3p_1}{(2\pi)^3}\frac{1}{2E_1}\frac{d^3p_2}{(2\pi)^3}\frac{1}{2E_2}\cdots\frac{d^3p_N}{(2\pi)^3}\frac{1}{2E_N}\times$$
$$(2\pi)^4\,\delta^4(P - \sum_i p_i).$$

phase space approach

PHASE SPACE DOMINANCE

$$\mathcal{Q}_N(M) = \frac{1}{2}\,\mathrm{Im}\left[\ln\left(1+\int d\phi_N\, i\mathcal{M}\right)\right]$$

- structureless scattering

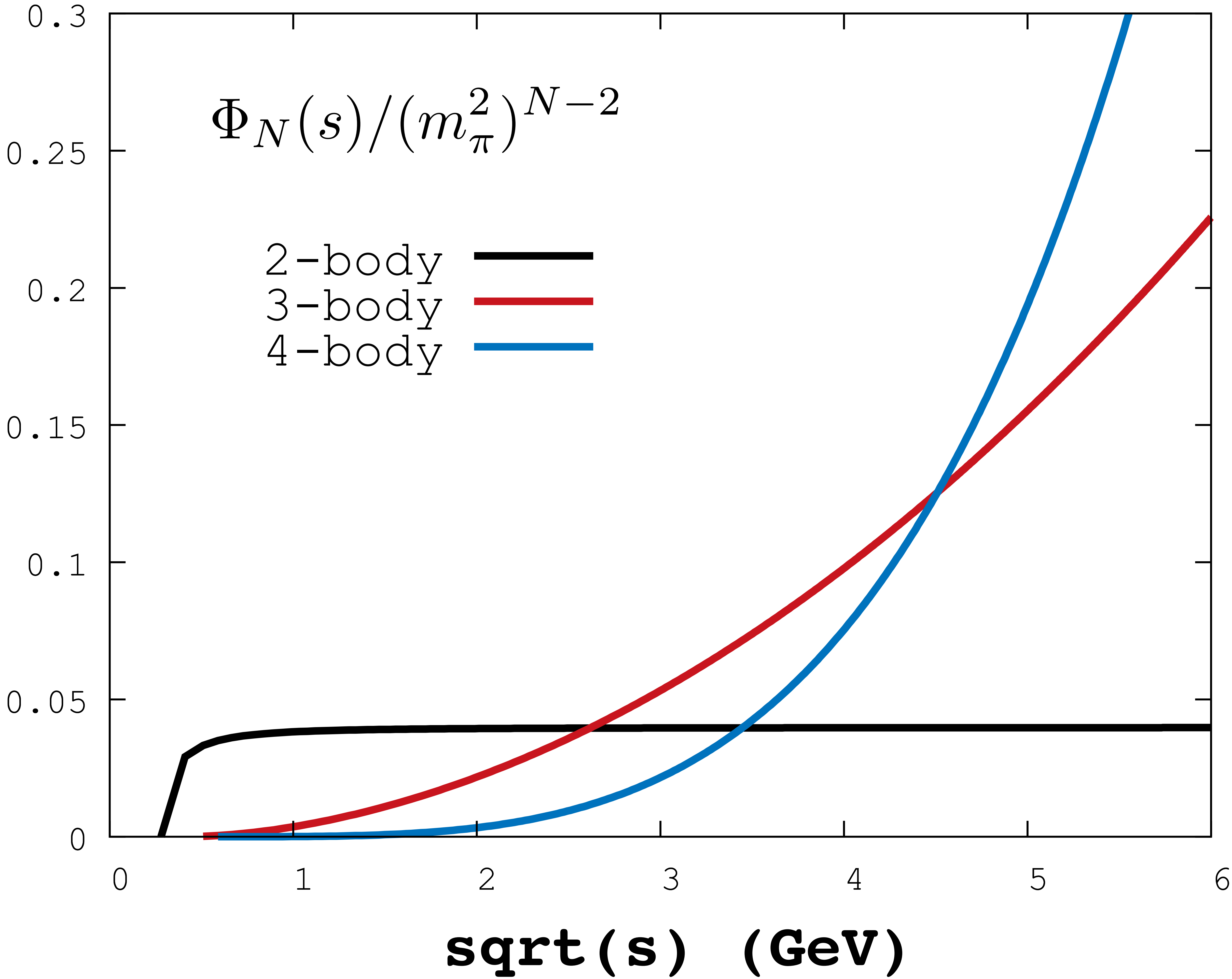
Dimension: $\sim E^{2N-4}$

$$i\mathcal{M} = i\,\lambda_N$$

Källén triangle function

$$\phi_N(s) = \frac{1}{16\,\pi^2 s}\int_{s'_-}^{s'_+} ds'\,\sqrt{\lambda(s,s',m_N^2)}\times$$

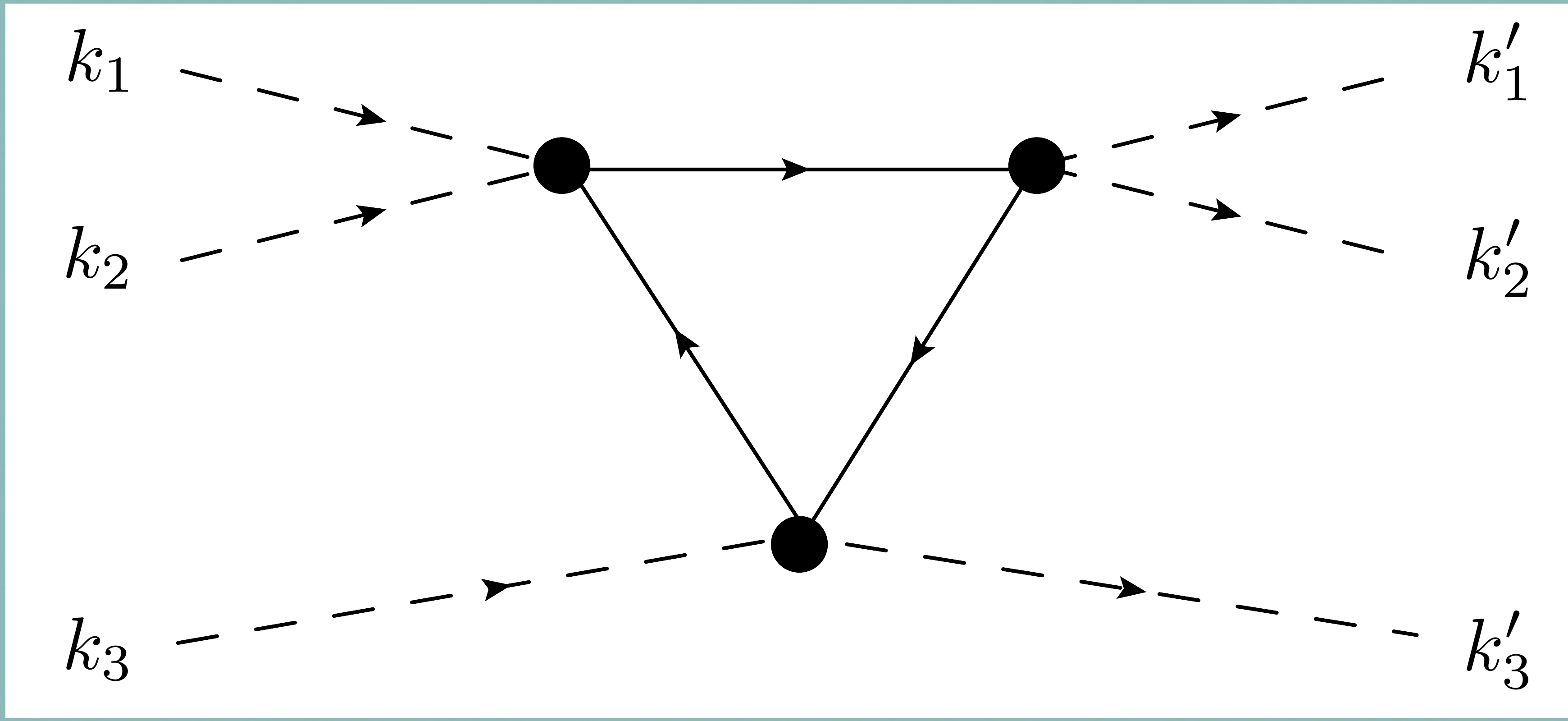
$$\phi_{N-1}(s',m_1^2,m_2^2,...,m_{N-1}^2)$$



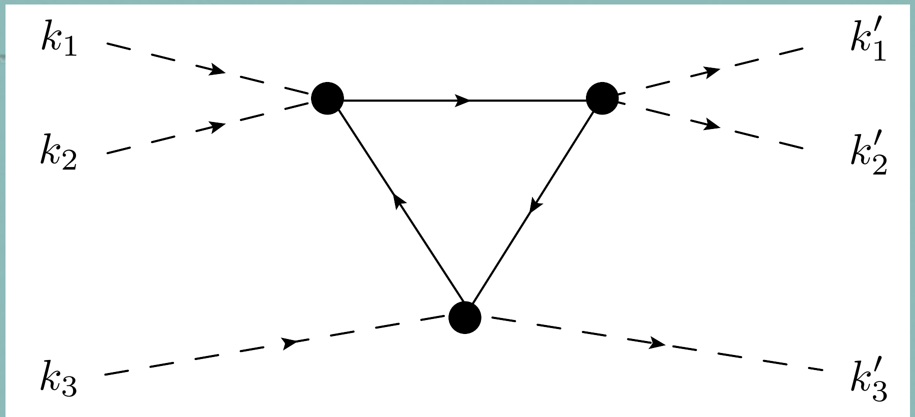
0.3
0.25
0.2
0.15
0.1
0.05
0
0
1
2
3
4
5
6
$\Phi_N(s)/(m_\pi^2)^{N-2}$
2-body
3-body
4-body
sqrt(s) (GeV)

TRIANGLE DIAGRAM

- 3-body diagram



Explicit calculation



$$i\mathcal{M}^{\triangle}(Q_1, Q_2, Q_3) = \int \frac{d^4 l}{(2\pi)^4} \times (-i\,\lambda)^3 \times i\,G(l) \times i\,G(l+Q_1) \times i\,G(l-Q_2)$$

Feynman's trick + dim reg.

$$i\mathcal{M}^{\triangle}(Q_1^2, Q_2^2, s = P_I^2) = -i\frac{\lambda^3}{16\pi^2}\int_0^1 dx \int_0^{1-x} dy\,\frac{1}{\Delta(x,y)}$$

$$\Delta(x,y) = m_\pi^2 - x(1-x)\,Q_1^2 - y(1-y)\,Q_2^2 - 2\,xy\,Q_1\cdot Q_2 - i\epsilon.$$

- to lowest order $\mathcal{Q}(s) \approx \frac{1}{2}\,\mathrm{Im}\left[\int d\phi_3\, i\mathcal{M}^{\mathrm{triangle}}\right],$

  => only need to deal with on-shell condition

  $$k_i' = k_i$$

  analytic result:

$$i\mathcal{M}^{\triangle,o.s.}(Q_1^2,s) = -i\frac{\lambda^3}{16\,\pi^2}\,\frac{z}{Q_1^2}\,\ln\frac{1-z}{1+z}$$

$$z = \frac{1}{\sqrt{1-\frac{4m_\pi^2}{Q_1^2}}}.$$

$$\mathcal{Q}(s) \approx \frac{1}{2}\,\mathrm{Im}\left[\int d\phi_3\, i\mathcal{M}^{\mathrm{triangle}}\right],$$

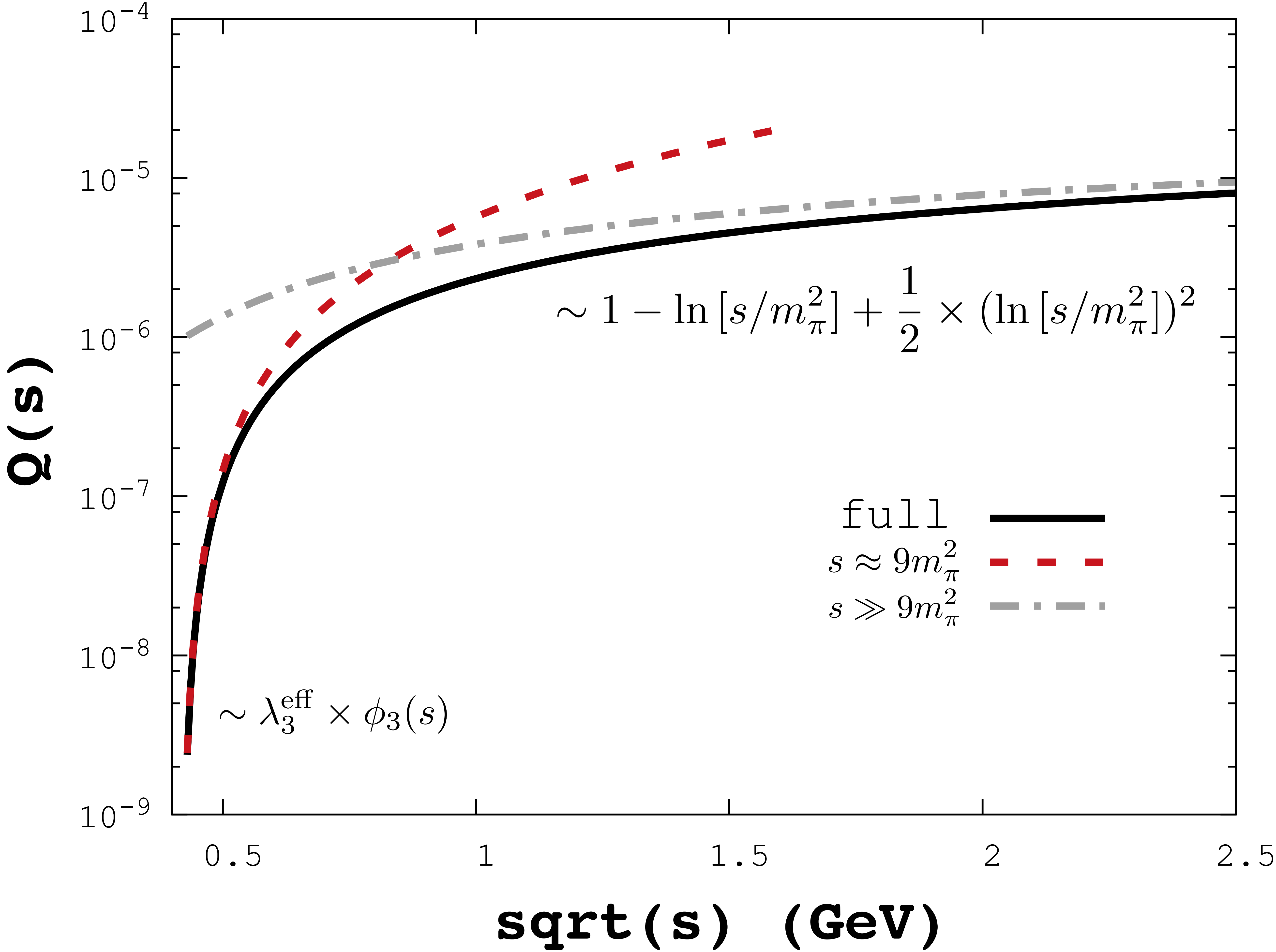
Limits:

$s \to 9m_\pi^2$ $$\mathcal{Q}(s) \approx \frac{1}{2} \times \lambda_3^{\mathrm{eff}} \times \phi_3(s).$$

$s \gg 9m_\pi^2$ $$\mathcal{Q}(s) \approx \frac{\lambda^3}{8192\,\pi^5} \int_{\xi_0}^{1} d\xi\,(\frac{1}{\xi} - 1)\,\left[-z\,\ln\left|\frac{1-z}{1+z}\right|\right]$$

$$\approx \frac{\lambda^3}{4096\,\pi^5} \times \left[1 + \ln\frac{\xi_0}{4} + (\ln\frac{\xi_0}{4})^2\right]$$

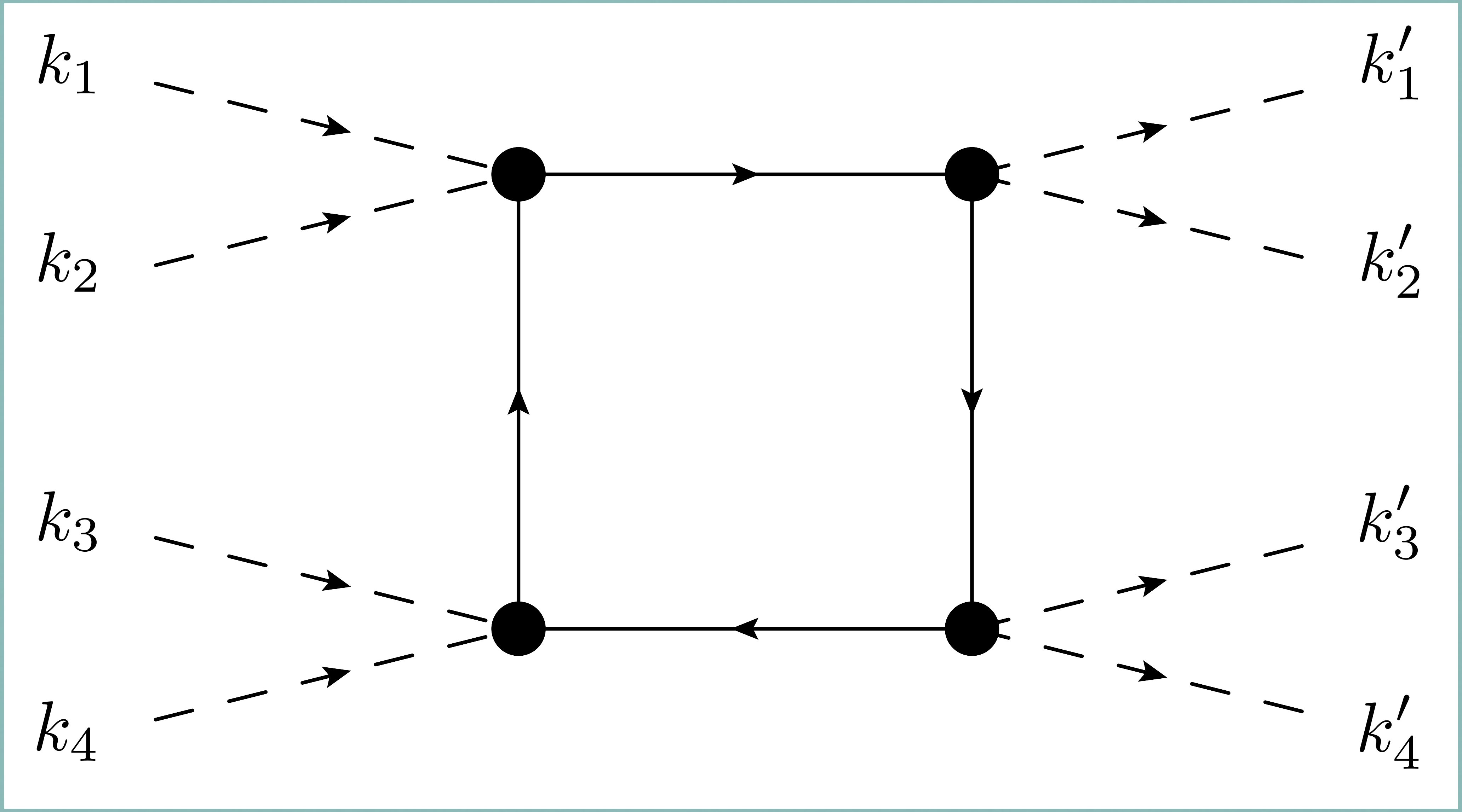
where $\xi_0 = \dfrac{4m_\pi^2}{s}.$

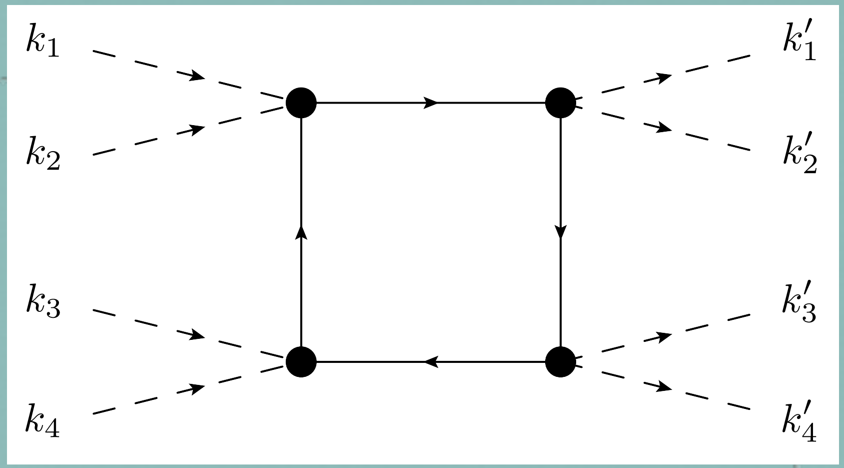


10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
Q(s)
0.5
1
1.5
2
2.5
sqrt(s) (GeV)
$\sim 1 - \ln[s/m_\pi^2] + \frac{1}{2} \times (\ln[s/m_\pi^2])^2$
$\sim \lambda_3^{\mathrm{eff}} \times \phi_3(s)$
full
$s \approx 9m_\pi^2$
$s \gg 9m_\pi^2$

BOX DIAGRAM

- 4-body diagram





Explicit calculation

$$i\mathcal{M}^{\rm box}(Q_1,Q_2,Q_3,Q_4) = \int \frac{d^4 l}{(2\pi)^4} (-i\,\lambda)^4 \times i\,G(l) \times i\,G(l+Q_1) \times i\,G(l+Q_1-Q_3) \times i\,G(l-Q_2)$$

Feynman's trick + dim reg.

$$i\mathcal{M}^{\rm box}(Q_1,Q_2,Q_3,Q_4) = i\frac{\lambda^4}{16\pi^2}\int_0^1 dx \int_0^{1-x} dy \times \int_0^{1-x-y} dz \times \left(\frac{1}{\Delta(x,y,z)}\right)^2$$

$$\mathcal{Q}(s) \approx \frac{1}{2}\,\mathrm{Im}\left[\int d\phi_4\, i\mathcal{M}^{\mathrm{box,o.s.}}\right].$$

Limits: $s \to 16\, m_\pi^2$

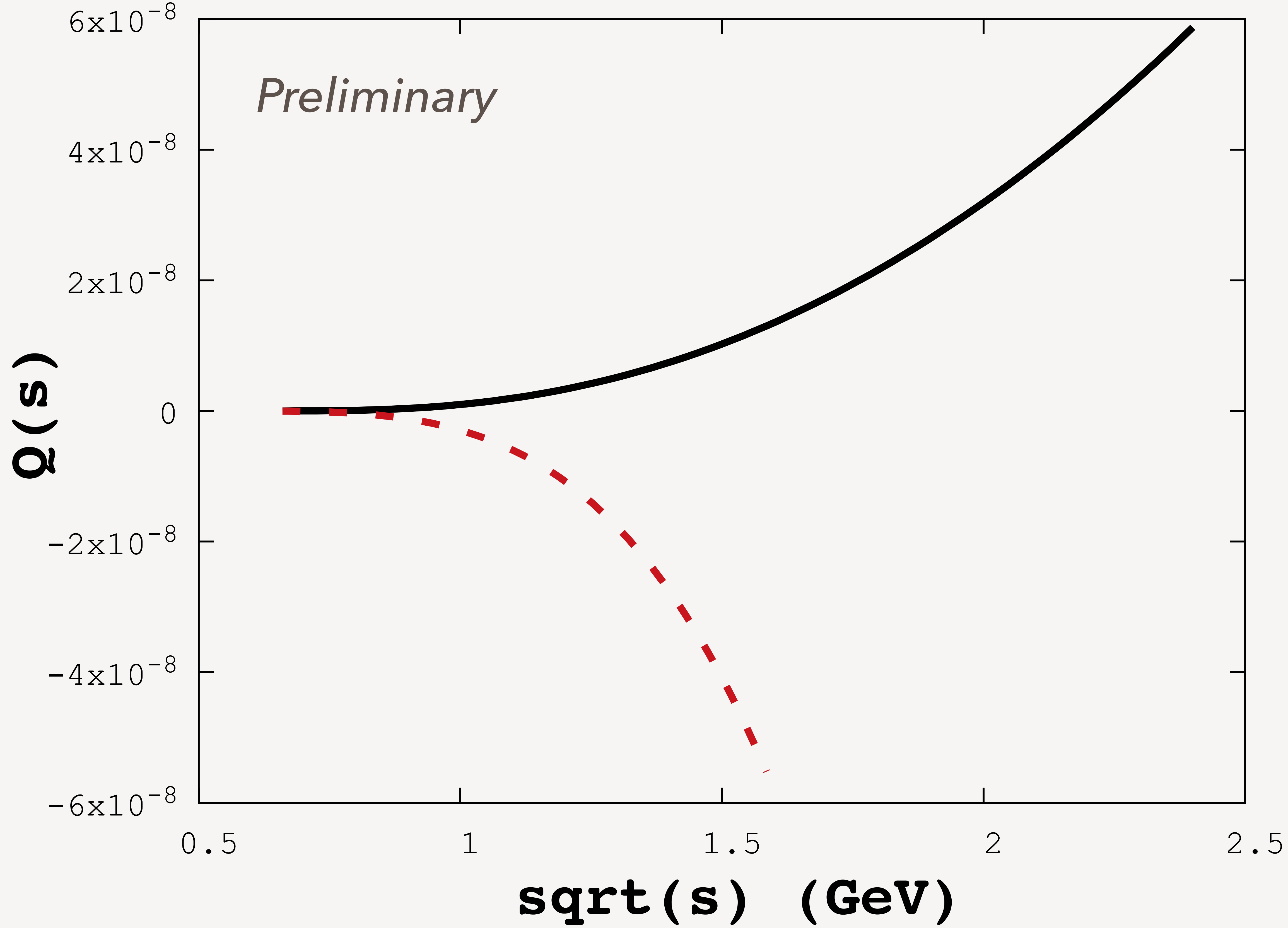
$$i\mathcal{M}^{\triangle,o.s.}(Q_1^2, s) \approx i\lambda_4^{\mathrm{eff}}$$

$$\lambda_4^{\mathrm{eff}} = \frac{\lambda^4}{256\pi^2}\frac{1}{m_\pi^4} \times (\frac{\sqrt{3}}{2}\ \ln(7 - 4\sqrt{3}) + 2)$$

$$\mathcal{Q}(s) \approx \frac{1}{2} \times \lambda_4^{\mathrm{eff}} \times \phi_4(s).$$

Negative!

$s >> 16\, m_\pi^2$ *???*


6x10⁻⁸
4x10⁻⁸
2x10⁻⁸
0
-2x10⁻⁸
-4x10⁻⁸
-6x10⁻⁸
Preliminary
Q(s)
0.5
1
1.5
2
2.5
sqrt(s) (GeV)

SUMMARY

- change in density of state / time delay due to interaction

$$2\frac{d\delta}{dE}$$

- S-matrix approach to thermodynamics
- Extend to N-body with phase space expansion

THANK YOU

完

BACKUP

APPLICATION
PION + NUCLEON + DELTA SYSTEM

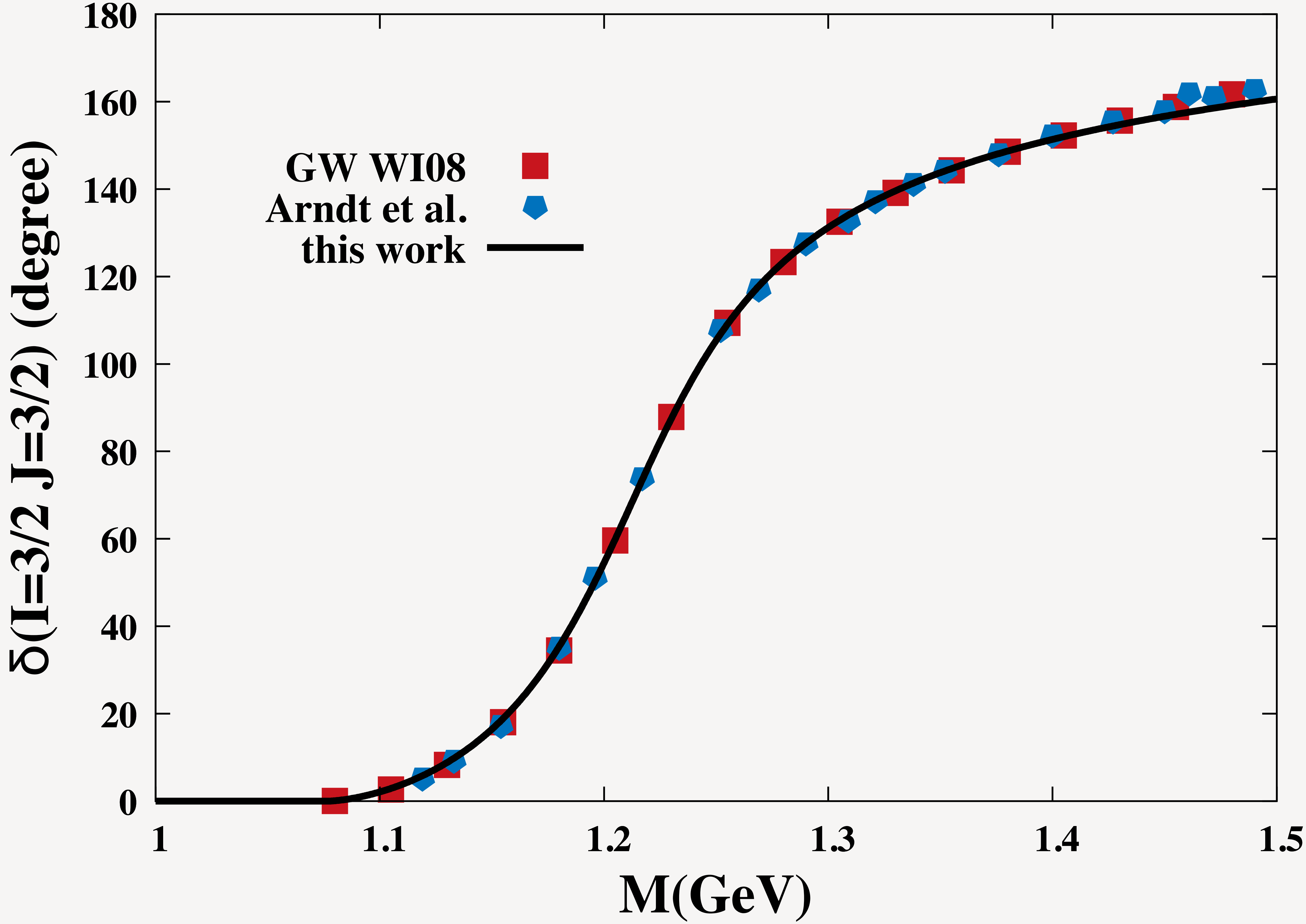
WHAT'S IN A NAME? THAT WHICH WE CALL A RESONANCES?

- A resonance is MORE than a MASS and a **WIDTH**

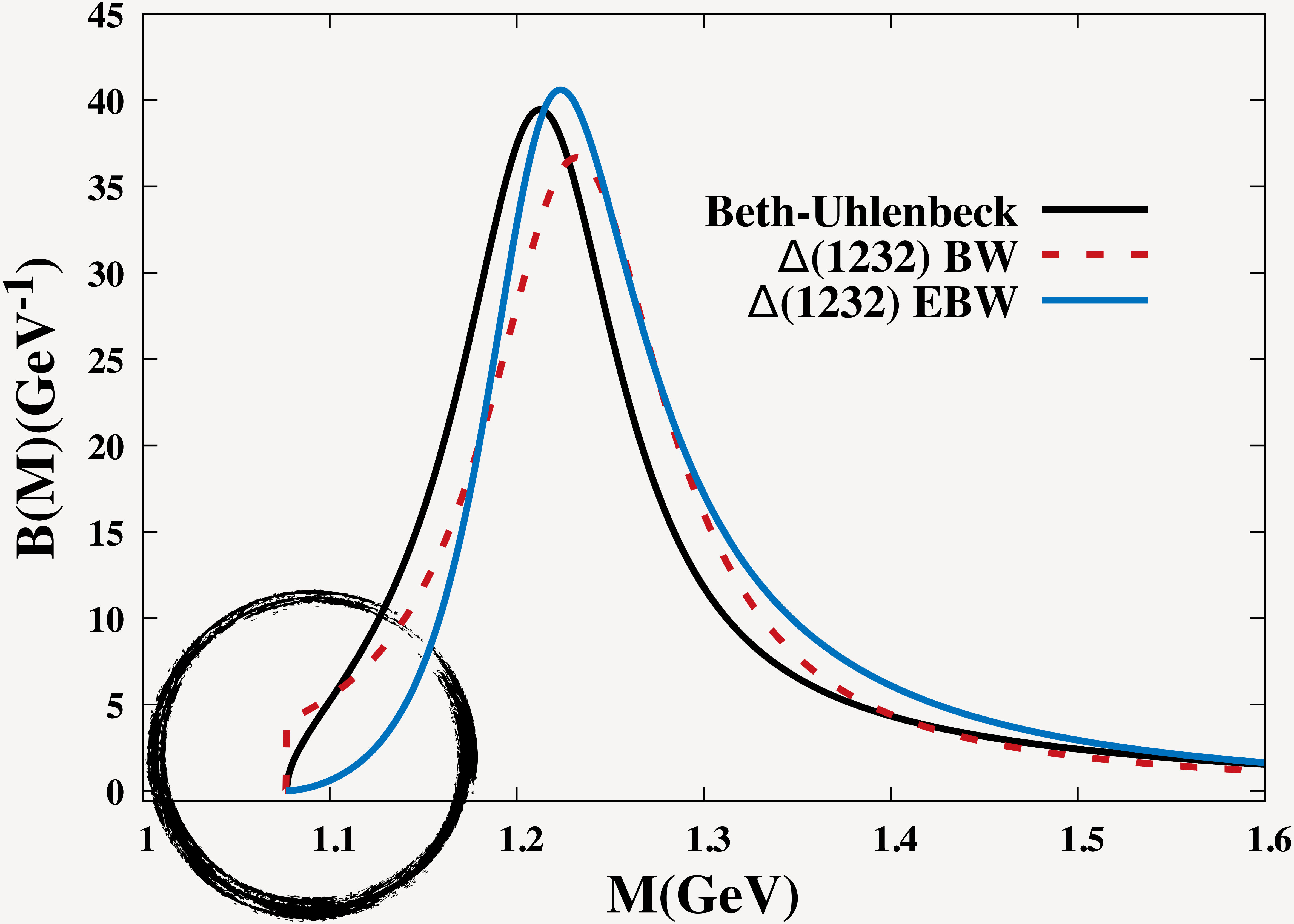
$\boxed{\Delta(1232)\ 3/2^+}$

$I(J^P) = \frac{3}{2}(\frac{3}{2}^+)$

Breit-Wigner mass (mixed charges) = 1230 to 1234 ($\approx$ 1232) MeV
Breit-Wigner full width (mixed charges) = 114 to 120 ($\approx$ 117) MeV
Re(pole position) = 1209 to 1211 ($\approx$ 1210) MeV
-2Im(pole position) = 98 to 102 ($\approx$ 100) MeV



GW WI08
Arndt et al.
this work
δ(I=3/2 J=3/2) (degree)
M(GeV)
0
20
40
60
80
100
120
140
160
180
1
1.1
1.2
1.3
1.4
1.5



B(M)(GeV-1)
M(GeV)
Beth-Uhlenbeck
Δ(1232) BW
Δ(1232) EBW
0
5
10
15
20
25
30
35
40
45
1
1.1
1.2
1.3
1.4
1.5
1.6

EFFECTIVE POTENTIAL FOR (PI N DELTA) SYSTEM

- Φ-derivable approach

$$\ln Z = -\frac{1}{2}\mathrm{Tr}\,\ln D_\pi^{-1} - \frac{1}{2}\mathrm{Tr}\,[\Pi_\pi D_\pi] + \mathrm{Tr}\,\ln G_N^{-1} + \mathrm{Tr}\,[\Sigma_N G_N] + \mathrm{Tr}\,\ln G_\Delta^{-1} + \mathrm{Tr}\,[\Sigma_\Delta G_\Delta] + \boxed{\Phi[D_\pi, G_N, G_\Delta]}$$

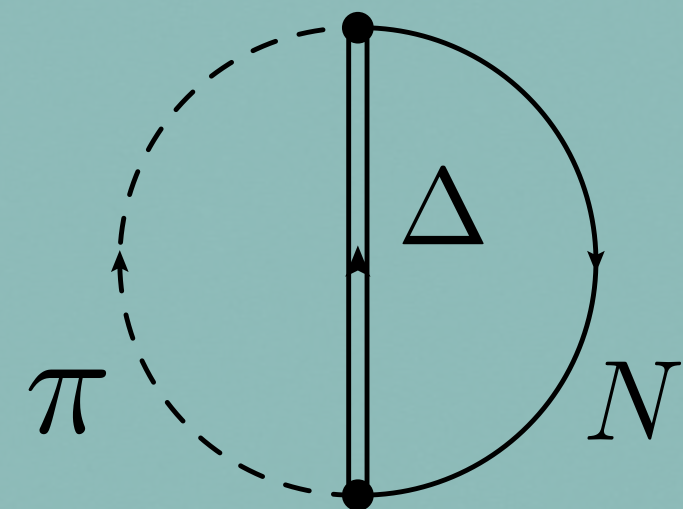
2PI

Gap equations

$$\frac{\partial \Phi}{\partial G_{(N,\Delta)}} = -\Sigma_{(N,\Delta)} \qquad \frac{\partial \Phi}{\partial D_\pi} = \Pi_\pi$$

BETH-UHLENBECK APPROXIMATION

$$\Phi \approx -\mathrm{Tr}\left[\Sigma_\Delta G_\Delta\right]$$



full GF!

$$\ln Z \approx \underbrace{-\frac{1}{2}\mathrm{Tr}\,\ln D^{(0)}{}_{\pi}^{-1} + \mathrm{Tr}\,\ln G^{(0)}{}_{N}^{-1}}_{\text{free gas}} + \underbrace{\mathrm{Tr}\,\ln G_\Delta^{-1}}_{\text{interaction}}$$

BETH-UHLENBECK APPROXIMATION

$$\delta = -\mathrm{Im}\ln G_\Delta^{-1}$$

$$B = 2\frac{\partial}{\partial E}\delta$$

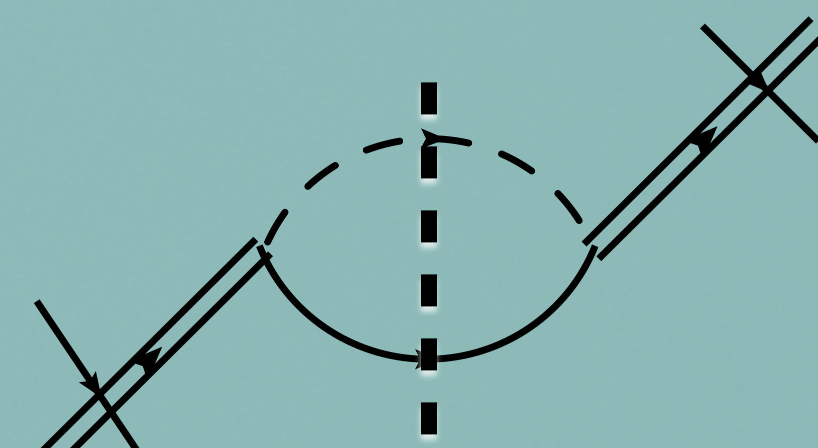
$$= -2\,\mathrm{Im}\,\frac{\partial}{\partial E}\ln G_\Delta^{-1}$$

$$= -2\,\mathrm{Im}\,[G_\Delta] + 2\,\mathrm{Im}\,[\frac{\partial \Sigma_\Delta}{\partial E} G_\Delta]$$

$$=> \boxed{\rho_\Delta(E) + \delta\rho_\Delta(E)}$$

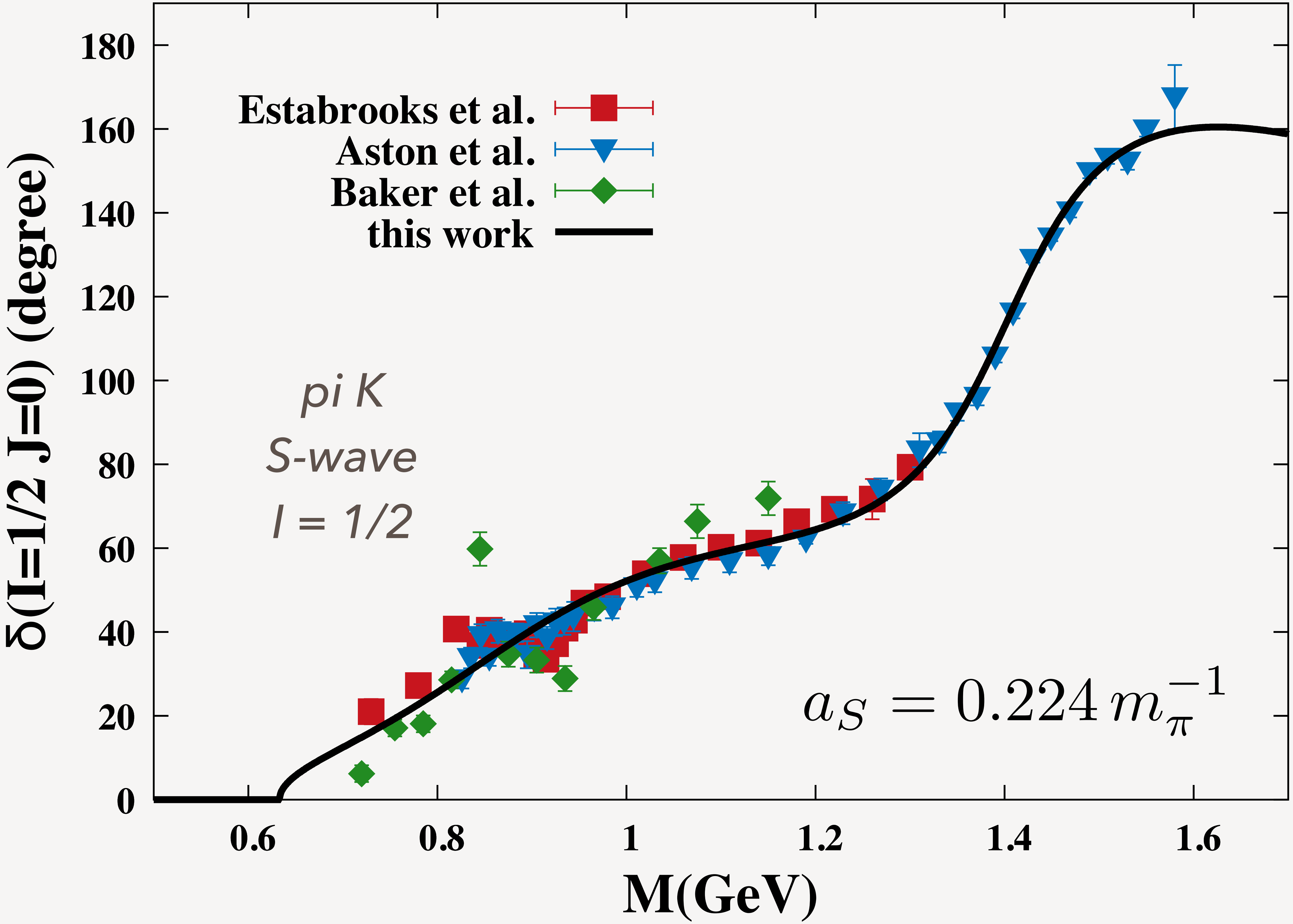
physical interpretation:

contribution from pi N pair

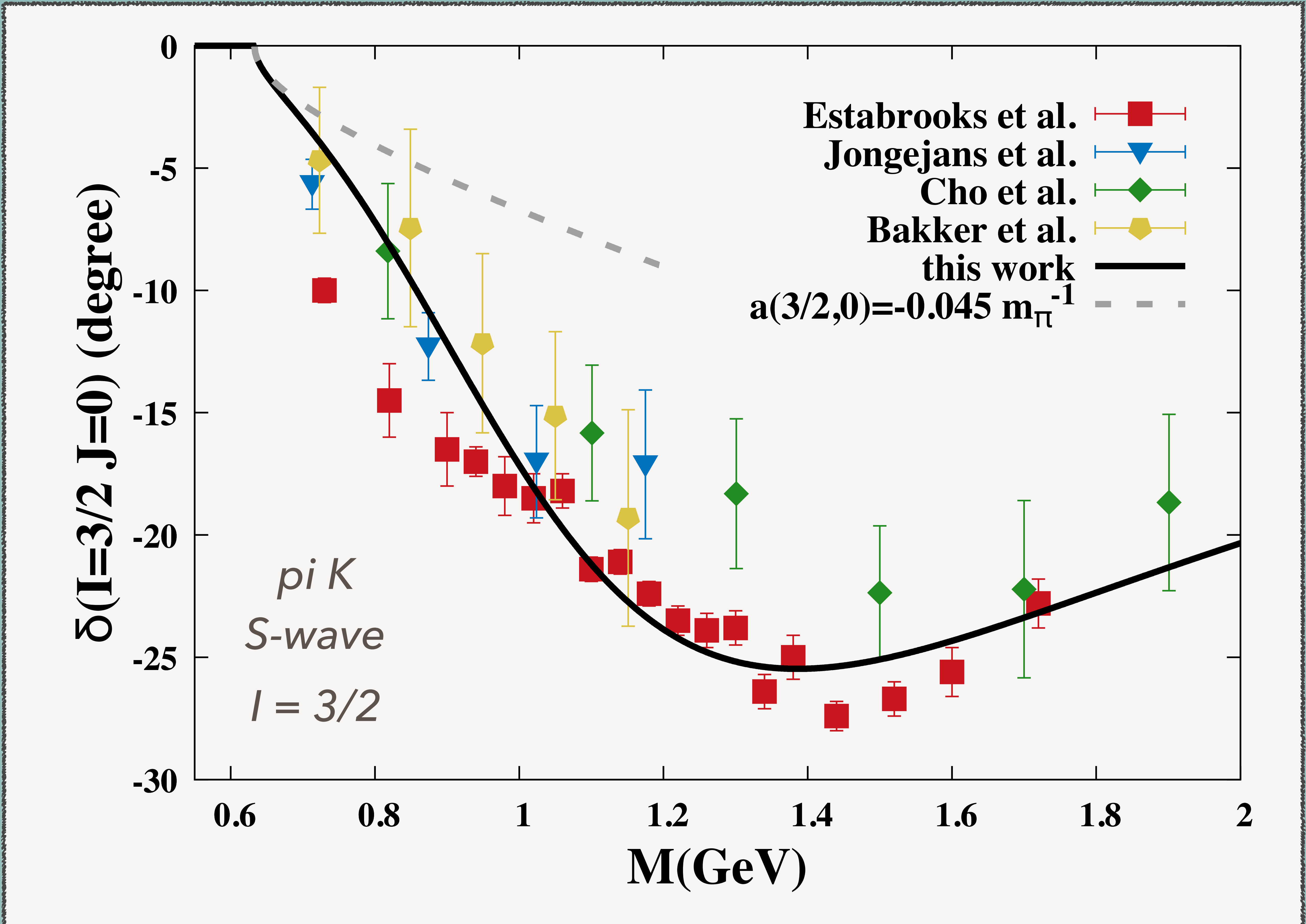


$$\frac{\partial \Sigma_\Delta}{\partial E}$$

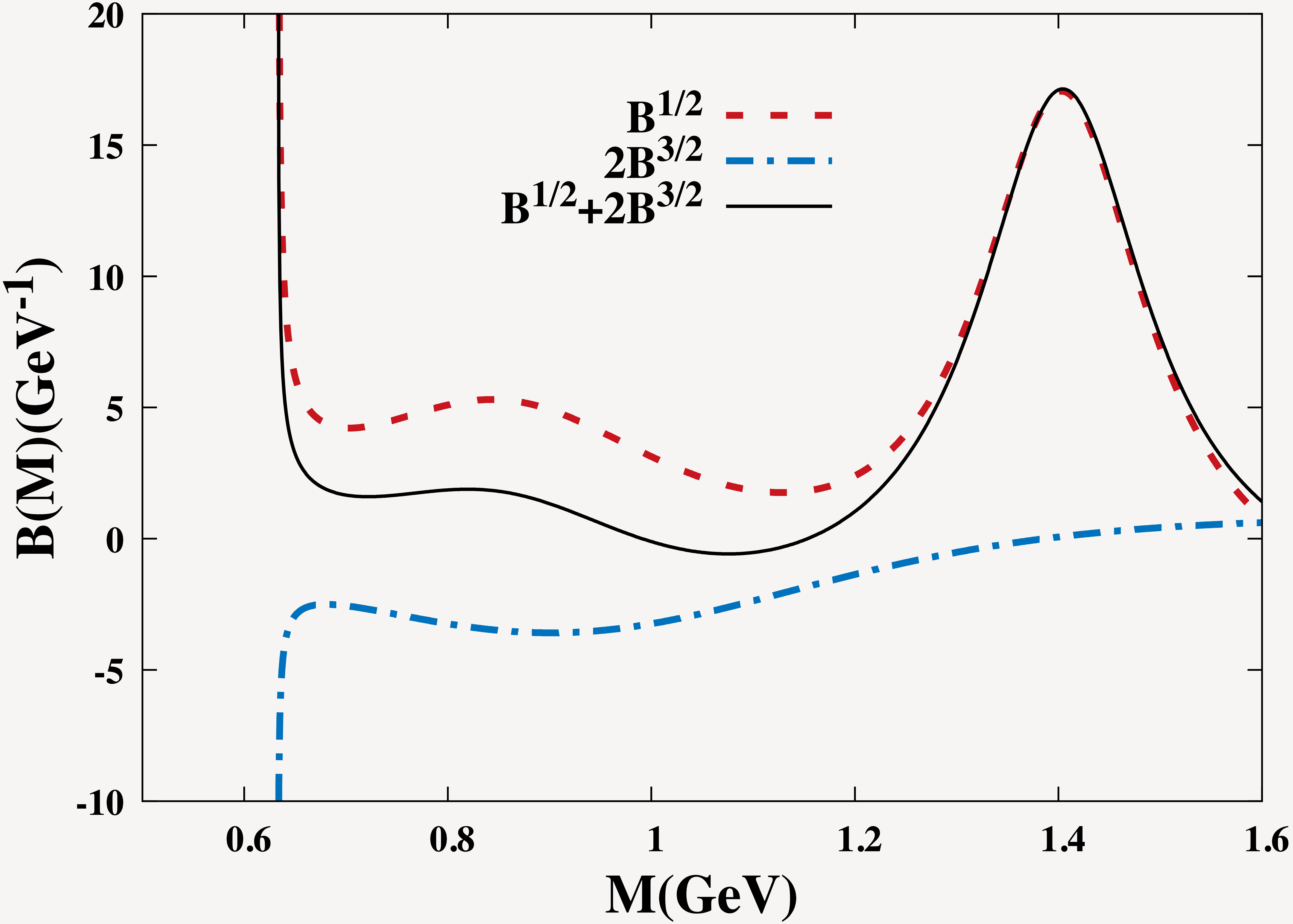
PI K SCATTERING



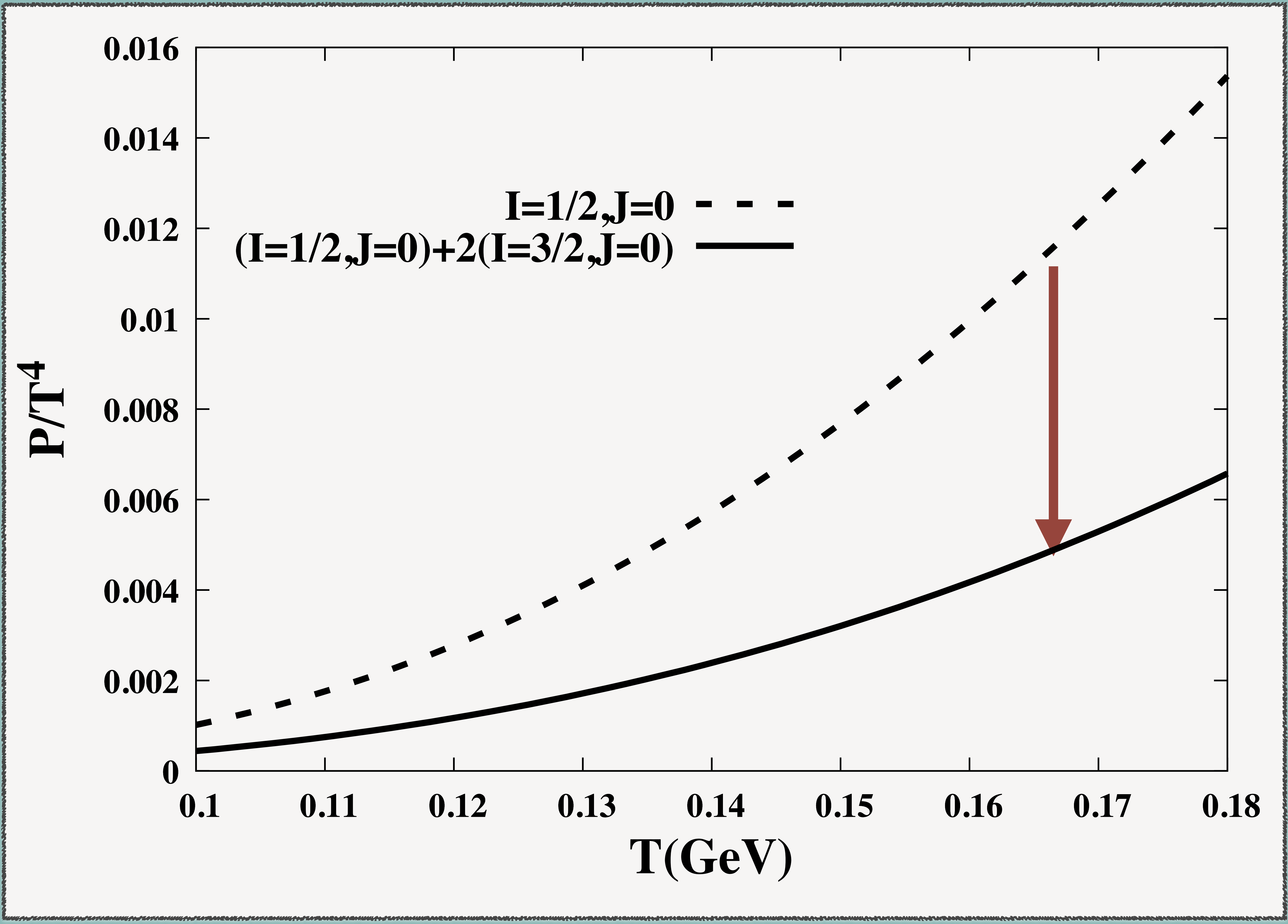
δ(I=1/2 J=0) (degree)
0
20
40
60
80
100
120
140
160
180
0.6
0.8
1
1.2
1.4
1.6
M(GeV)
Estabrooks et al.
Aston et al.
Baker et al.
this work
pi K
S-wave
I = 1/2
$a_S = 0.224\, m_\pi^{-1}$



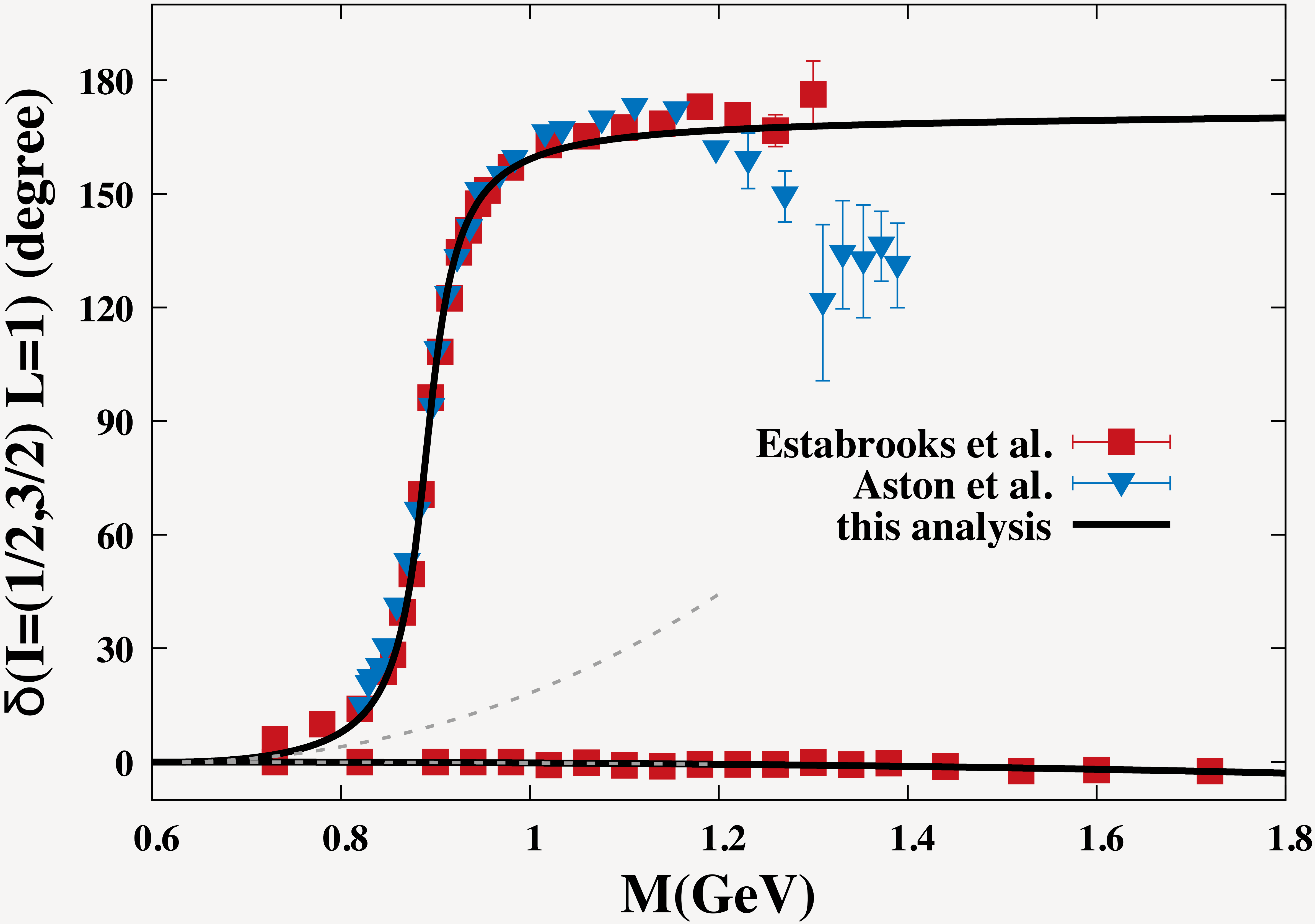
δ(I=3/2 J=0) (degree)
M(GeV)
0
-5
-10
-15
-20
-25
-30
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Estabrooks et al.
Jongejans et al.
Cho et al.
Bakker et al.
this work
a(3/2,0)=-0.045 m_π^{-1}
pi K
S-wave
I = 3/2

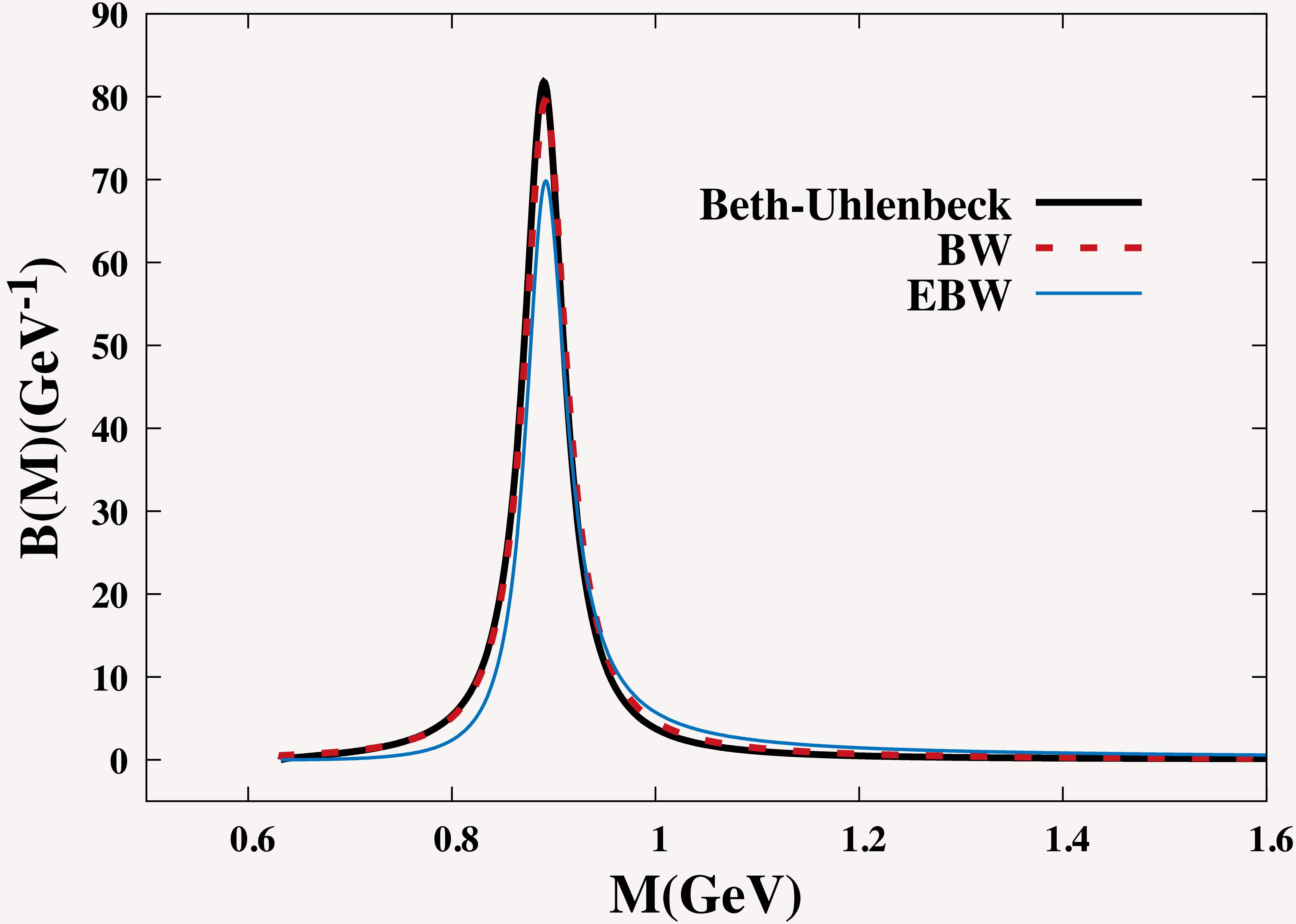


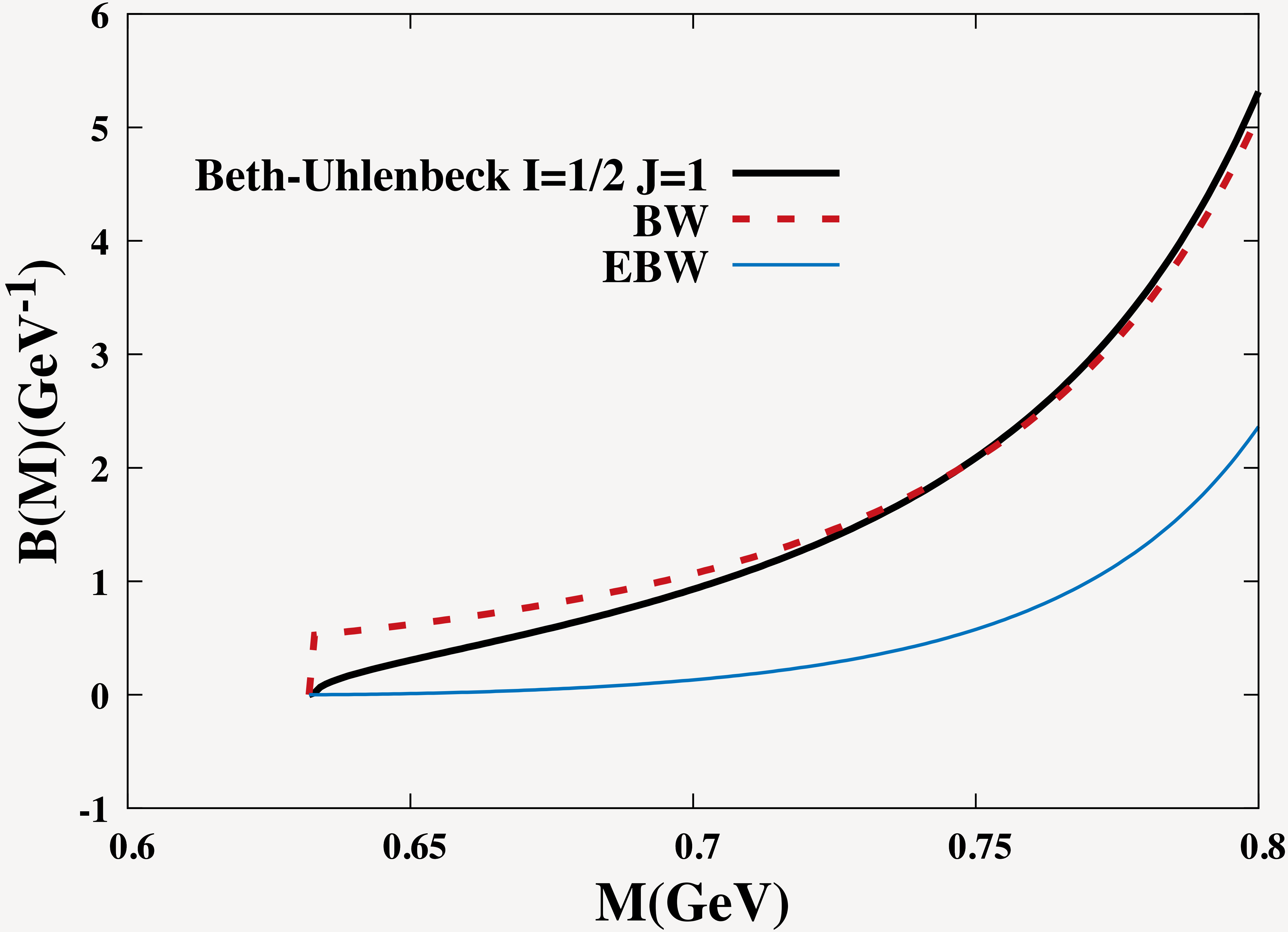
B(M)(GeV-1)
M(GeV)
B1/2
2B3/2
B1/2+2B3/2
20
15
10
5
0
-5
-10
0.6
0.8
1
1.2
1.4
1.6

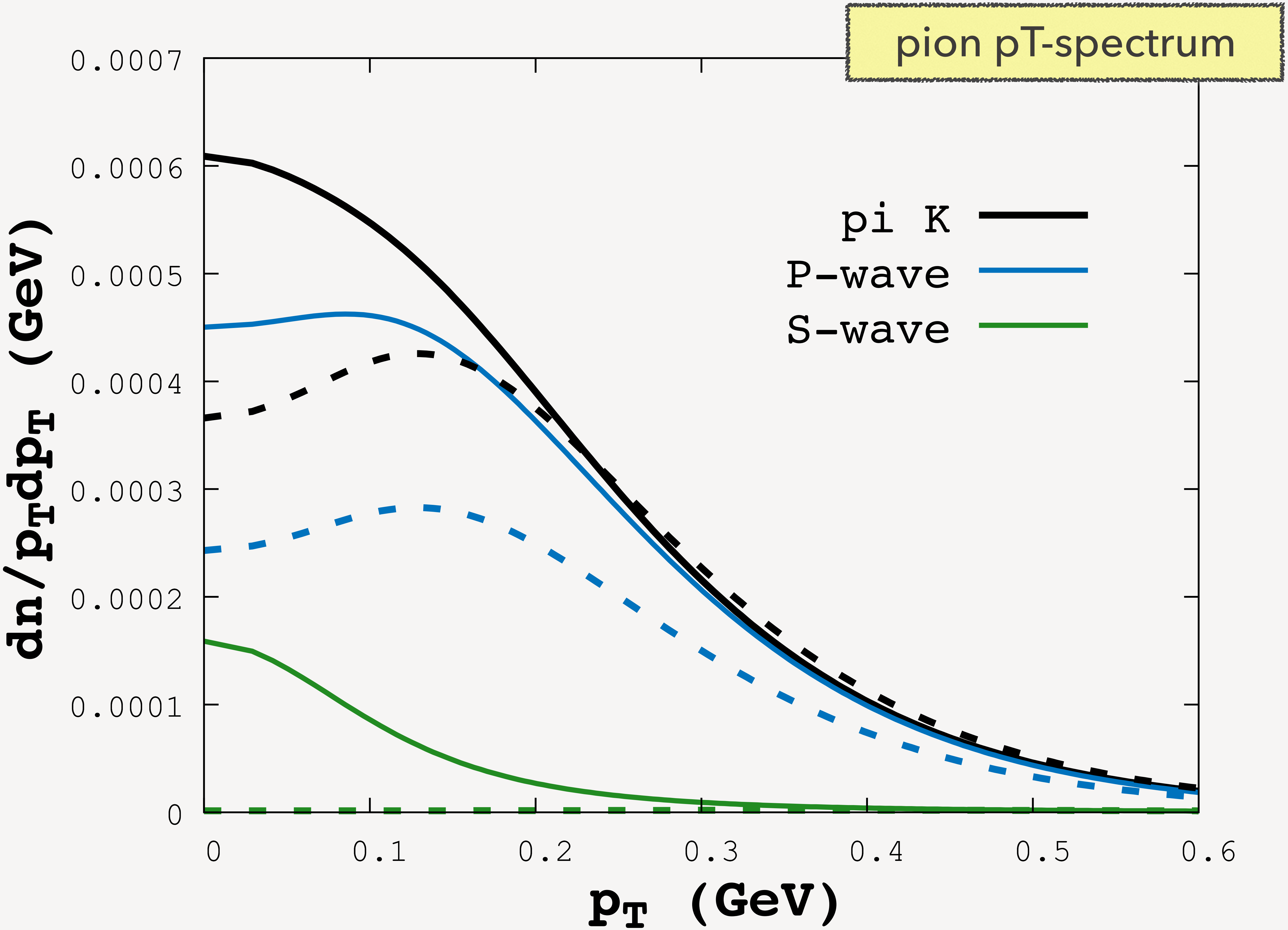


I=1/2,J=0
(I=1/2,J=0)+2(I=3/2,J=0)
P/T^4
T(GeV)
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18


δ(I=(1/2,3/2) L=1) (degree)
180
150
120
90
60
30
0
0.6
0.8
1
1.2
1.4
1.6
1.8
M(GeV)
Estabrooks et al.
Aston et al.
this analysis


B(M)(GeV-1)
M(GeV)
Beth-Uhlenbeck
BW
EBW
0
10
20
30
40
50
60
70
80
90
0.6
0.8
1
1.2
1.4
1.6


Beth-Uhlenbeck I=1/2 J=1
BW
EBW
B(M)(GeV-1)
M(GeV)
6
5
4
3
2
1
0
-1
0.6
0.65
0.7
0.75
0.8



pion pT-spectrum
pi K
P-wave
S-wave
0.0007
0.0006
0.0005
0.0004
0.0003
0.0002
0.0001
0
0.1
0.2
0.3
0.4
0.5
0.6
dn/p_Tdp_T (GeV)
p_T (GeV)